UNIVERSITY OF TWENTE.

Formal Methods & Tools.

Confluence versus Ample Sets in Probabilistic Branching Time

Mark Timmer

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YR-CONCUR 2011

Joint work with Henri Hansen

The context – probabilistic model checking

Probabilistic model checking:

- Verifying quantitative properties,
- Using a probabilistic model (e.g., an MDP)

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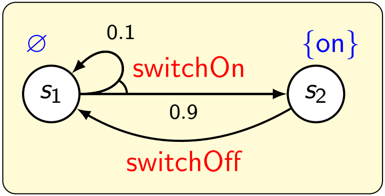

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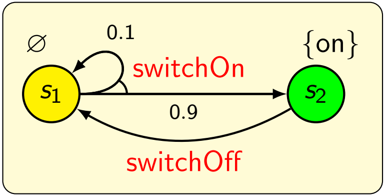

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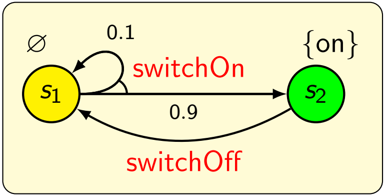

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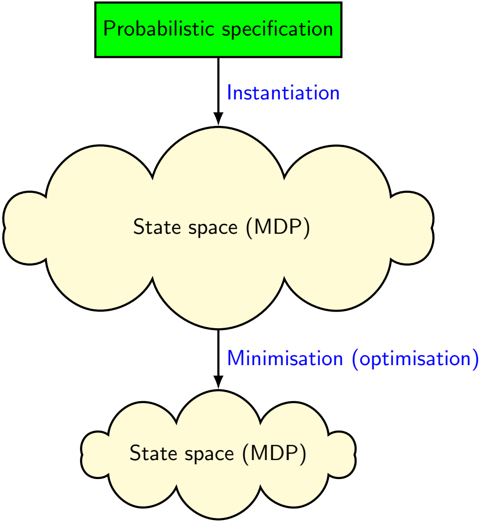
Main limitation (as for non-probabilistic model checking):

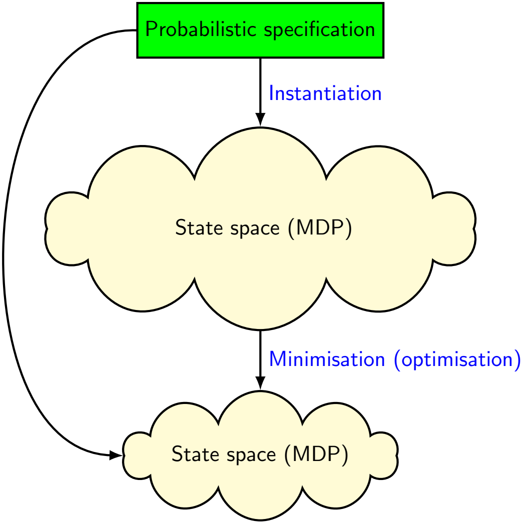
- Susceptible to the state space explosion problem

Combating the state space explosion



Combating the state space explosion

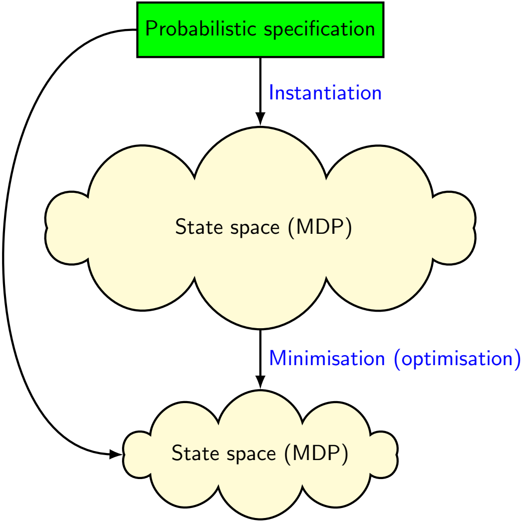
Optimised instantiation



Combating the state space explosion

Optimised instantiation

- Partial-order reduction
- Confluence reduction
  (initially for PAs)



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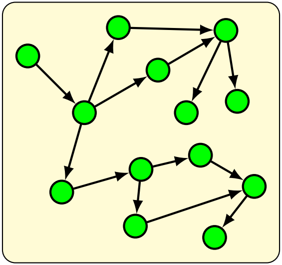

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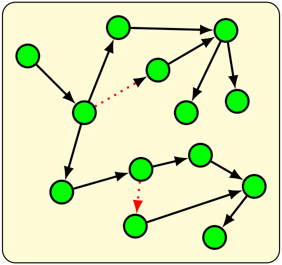

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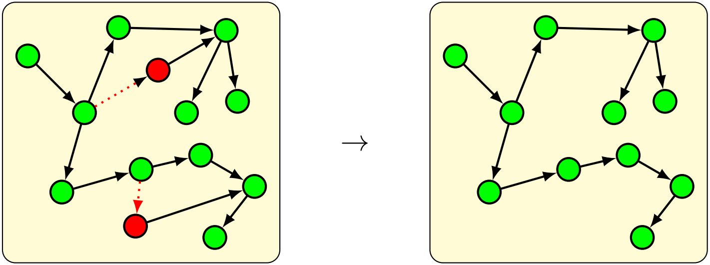

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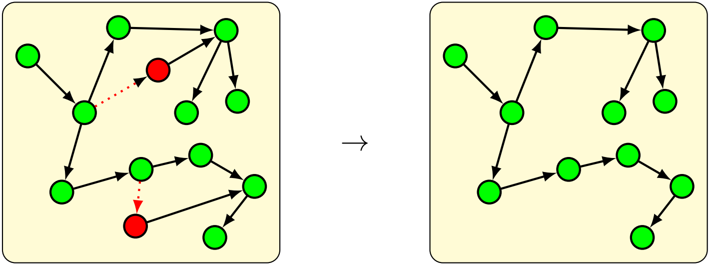

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$R\colon S \to 2^{\Sigma}$

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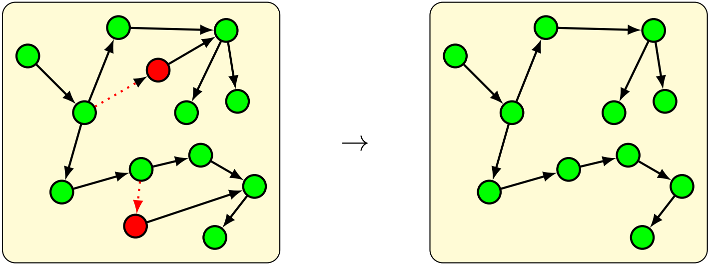

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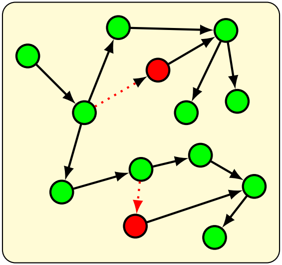

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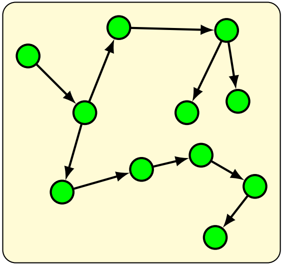
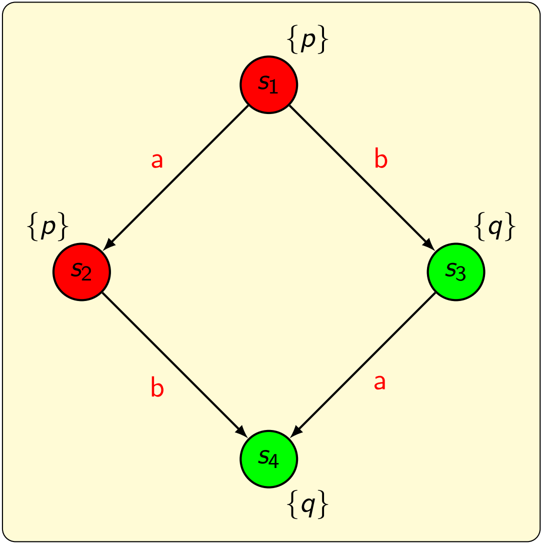


Reduction function:

$R\colon S \rightarrow 2^{\Sigma}$ ($R(s) \subseteq \text{enabled}(s)$)

If $R(s) \neq \text{enabled}(s)$, then $R(s)$ consists of reduction transitions.

Basic concepts



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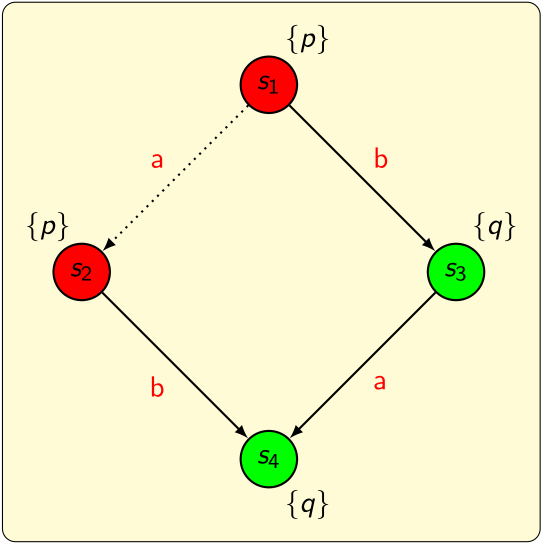

Stuttering transition:

- No observable change

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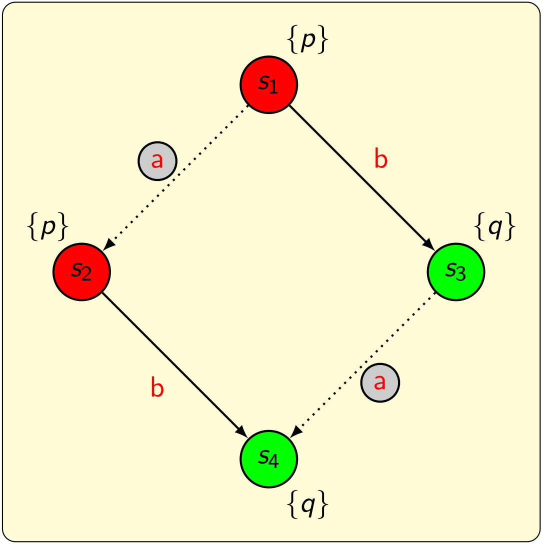

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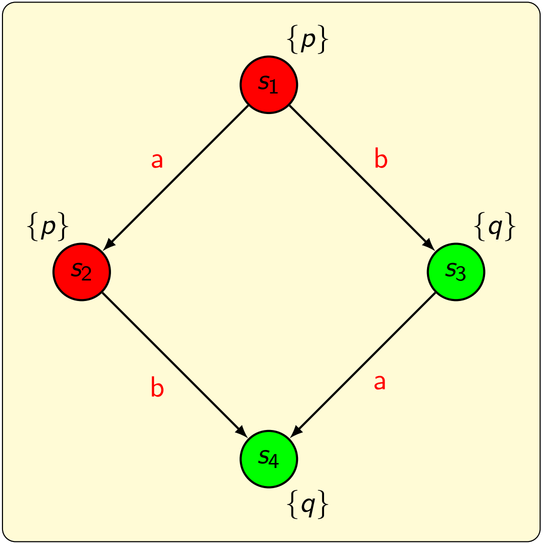

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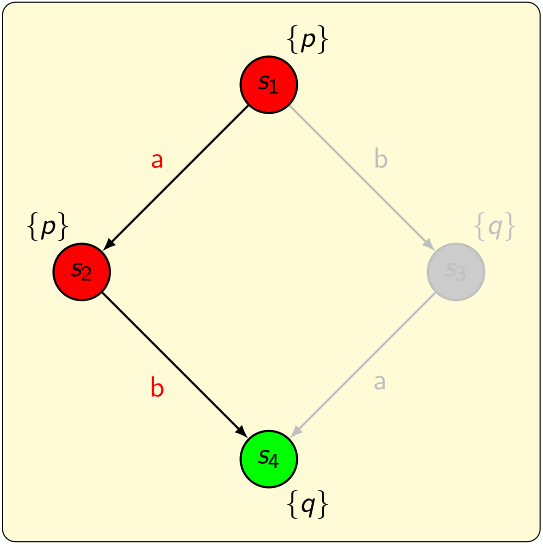

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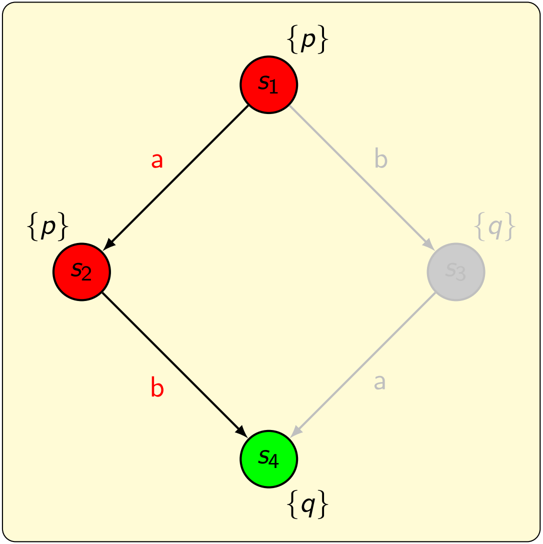

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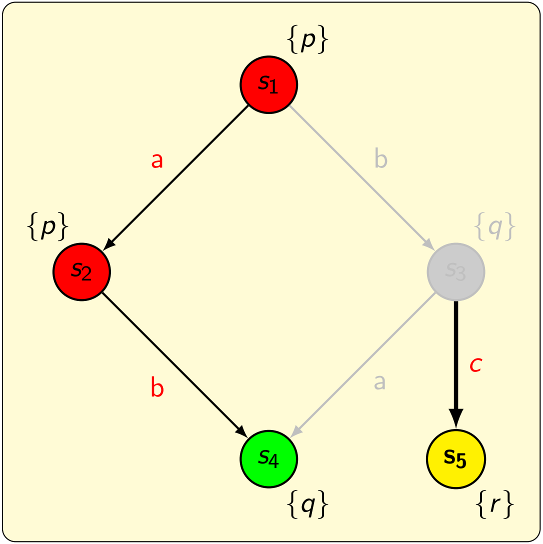

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- Preservation of $\text{LTL}_{\setminus X}$ (linear time)
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Linear time	[BGC'04, AN'04]	–
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- Based on independent actions and ample sets

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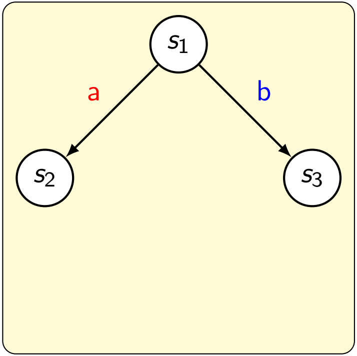

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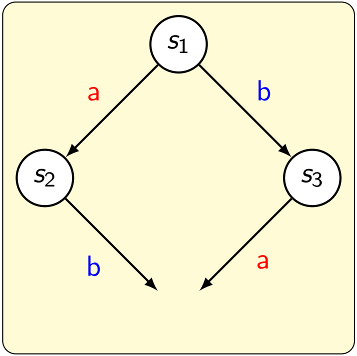

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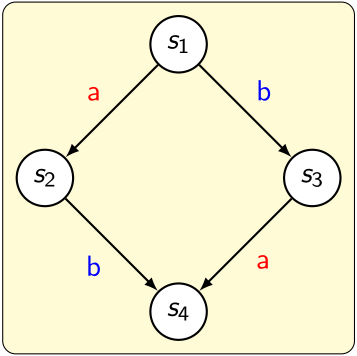

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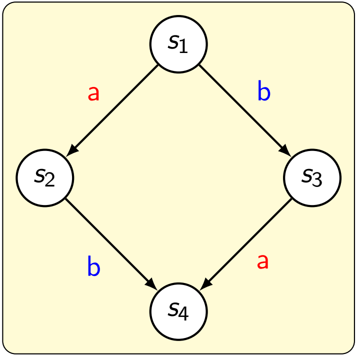

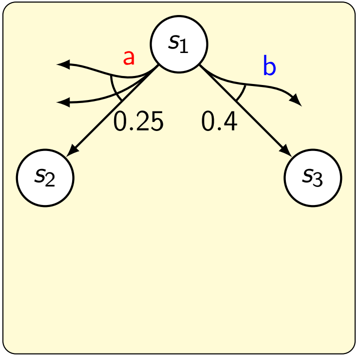

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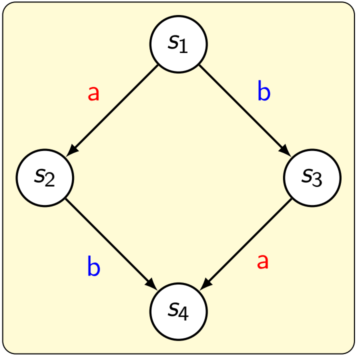

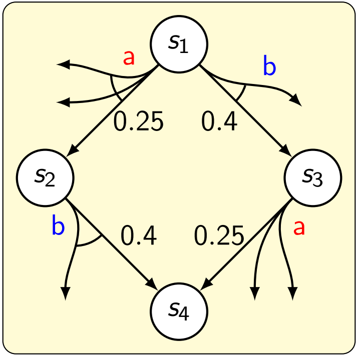

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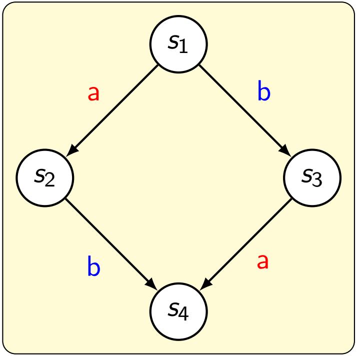

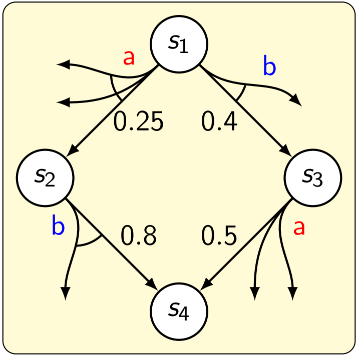

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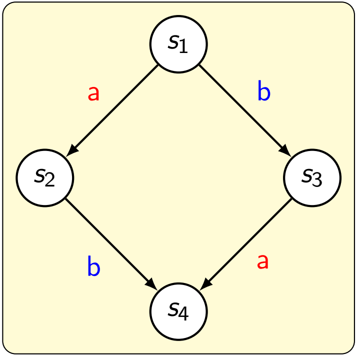

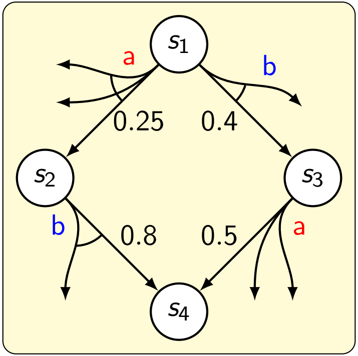

$$\mathbb{P}[s_1 \xrightarrow{ab} s] = \mathbb{P}[s_1 \xrightarrow{ba} s], \forall s$$

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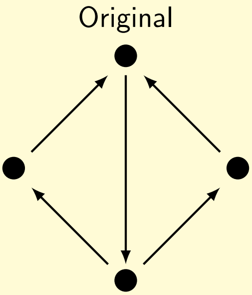

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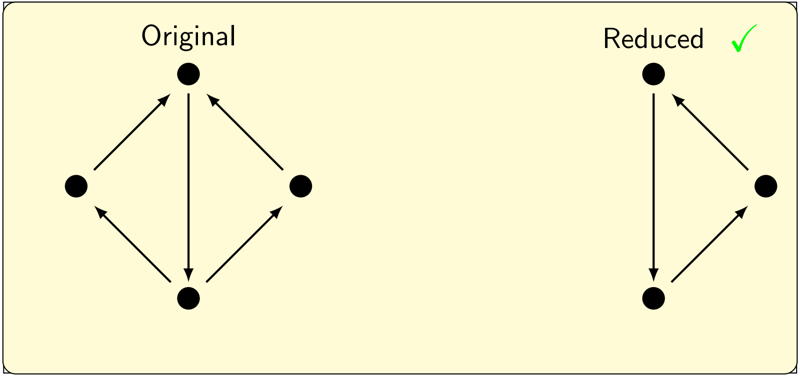

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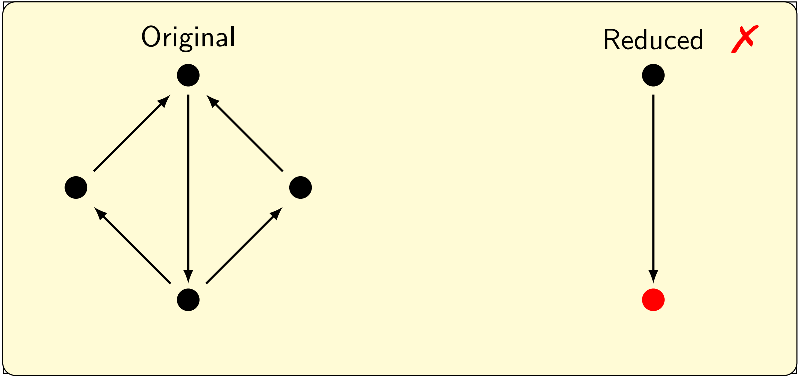

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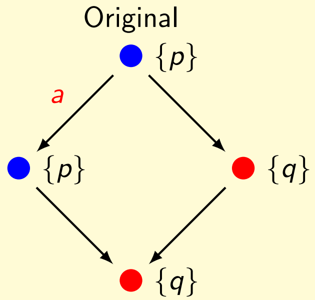

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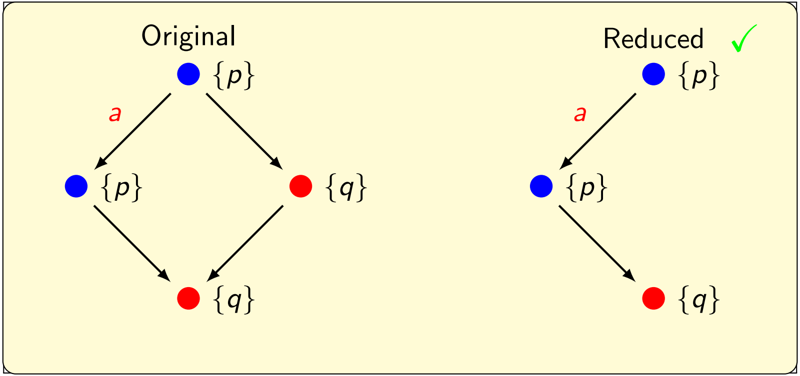

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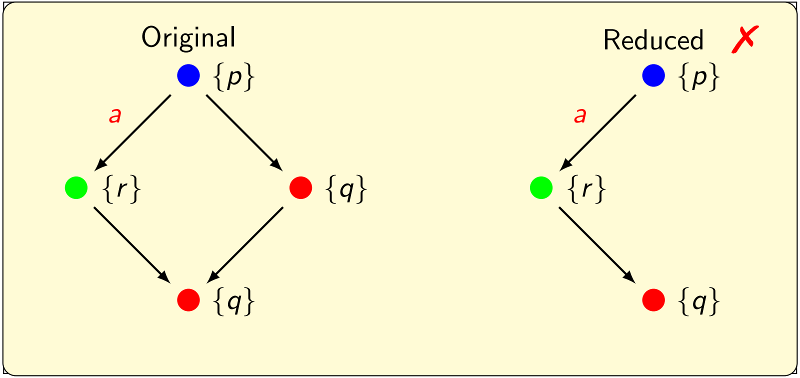

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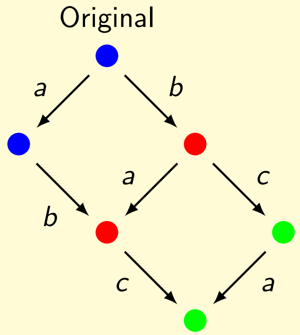

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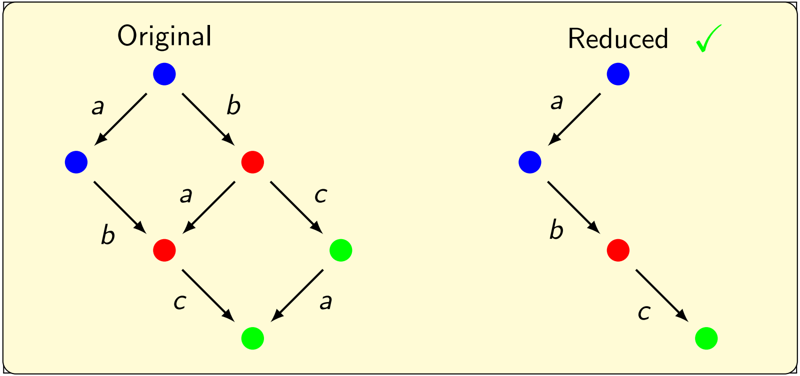

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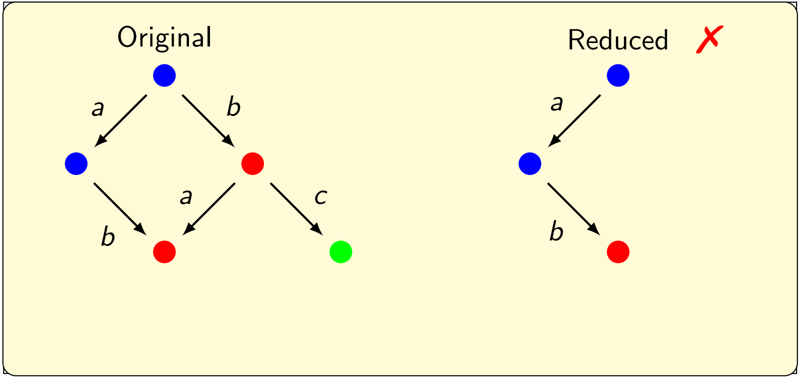

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A3 Every cycle in the reduced MDP contains a fully-expanded state (if $s \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_n} s_n = s$, then $\exists s_i \,.\, R(s_i) = \text{enabled}(s_i)$)

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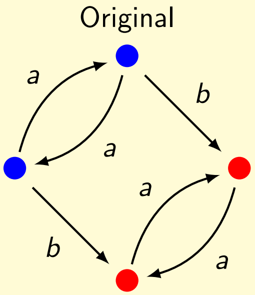

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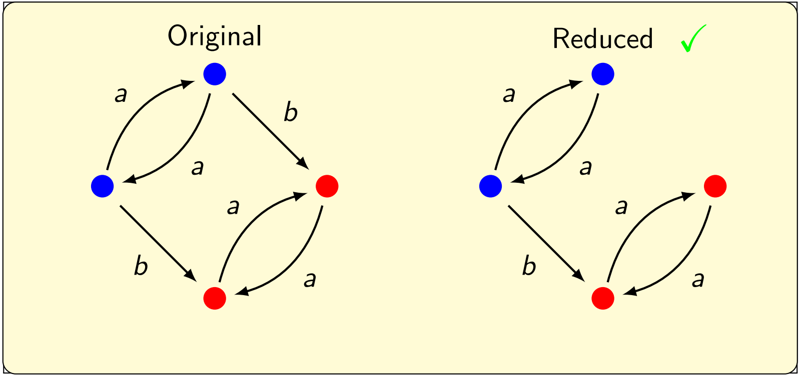

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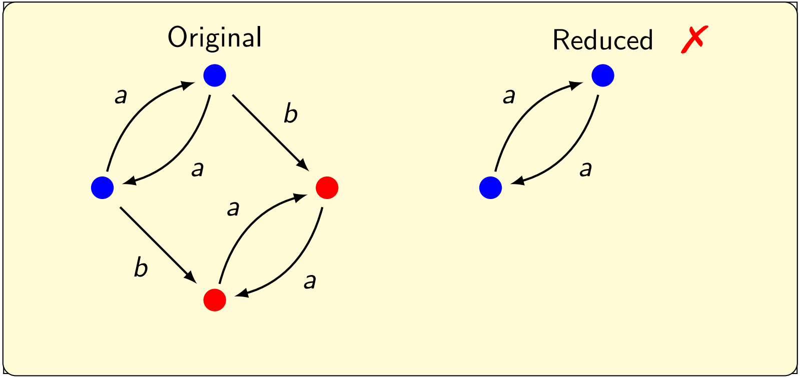

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- A4 if $R(s) \neq \text{enabled}(s)$, then $|R(s)| = 1$ and the chosen action is deterministic

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A3 Every cycle in the reduced MDP contains a fully-expanded state (if $s \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_n} s_n = s$, then $\exists s_i \,.\, R(s_i) = \text{enabled}(s_i)$)

A4 if $R(s) \neq \text{enabled}(s)$, then $|R(s)| = 1$ and the chosen action is deterministic

Partial-order reduction: ample sets

Partial-order reduction [Baier, D'Argenio, Größer, 2005]

- Based on independent actions and ample sets

Ample set conditions:

Given a reduction function $R\colon S \to 2^{\Sigma}$, for every $s \in S$

- **A0** $\varnothing \neq R(s)$ and $R(s) \subseteq \text{enabled}(s)$
- **A1** if $R(s) \neq \text{enabled}(s)$, then $R(s)$ contains only stuttering actions
- **A2** For every original path $s \xrightarrow{a_1} s_1 \xrightarrow{a_2} \dots \xrightarrow{a_n} s_n \xrightarrow{b} t$ such that $b \notin R(s)$ and b depends on $R(s)$, there exists an i such that $a_i \in R(s)$
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Confluence

Confluence reduction [Timmer, Stoelinga, van de Pol, 2011]

- Based on equivalent distributions and confluent transitions

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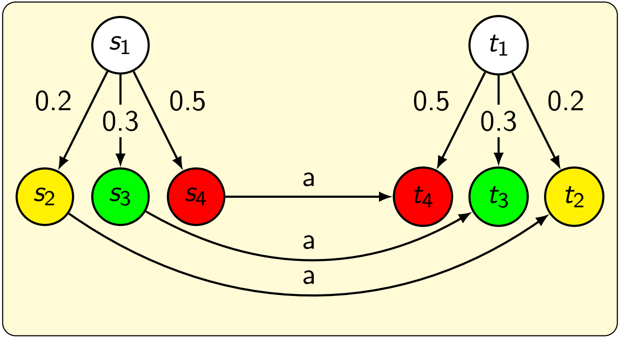

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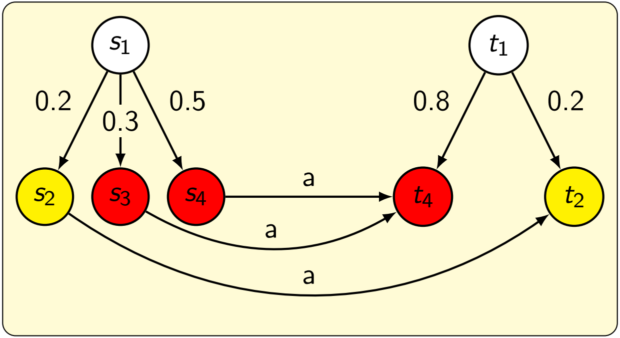

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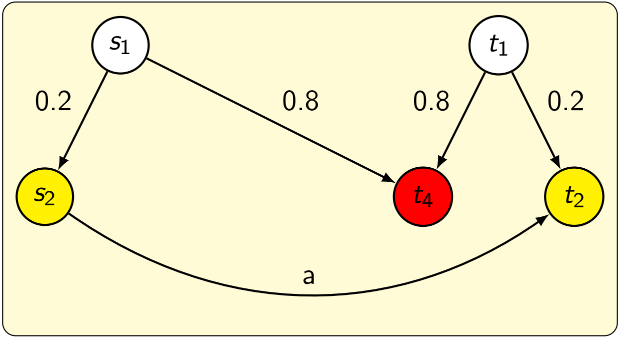

Confluence

Confluence reduction [Timmer, Stoelinga, van de Pol, 2011]

- Based on equivalent distributions and confluent transitions

The main idea:

- Choose a set T of transitions
- Make sure all of them are confluent
- $R(s) = \text{enabled}(s)$ or $R(s) = \{a\}$ such that $s \xrightarrow{a} t \in T$

Confluence

A set of deterministic and stuttering transitions T is confluent if

- If $s \xrightarrow{\tau} s' \in T$ and $s \xrightarrow{b} \mu$, then
  1. either $s' \xrightarrow{b} \nu$ and μ is T-equivalent to ν
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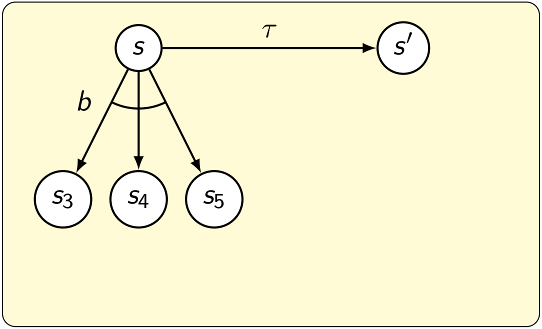

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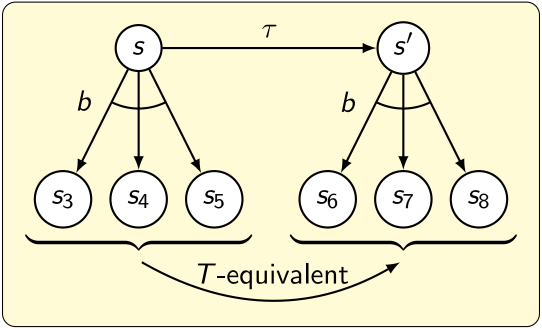

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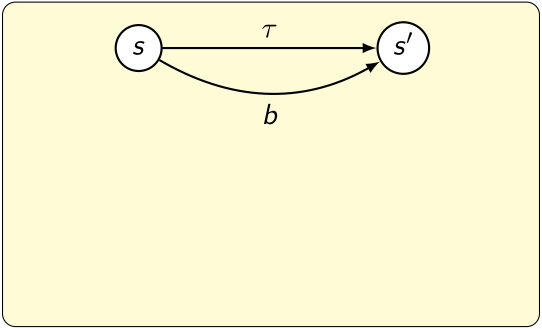

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Similarities among ample sets and confluence:

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Differences between ample sets and confluence:

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Let R be a reduction function satisfying the ample set conditions. Then, all reduction transitions are confluent.

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Let R be a reduction function satisfying the ample set conditions. Then, all reduction transitions are confluent.

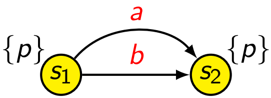
Or:

Any reduction allowed by partial-order reduction is also allowed by confluence reduction.

Proof (sketch).

1. Take the set of all reduction transitions of the partial-order reduction.
2. Recursively add transitions needed to complete the confluence diamonds.
3. Proof that the resulting set is indeed confluent. □

Comparison – Confluence does not imply POR



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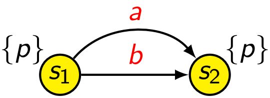

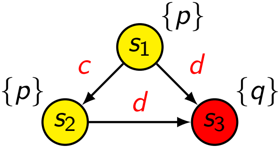

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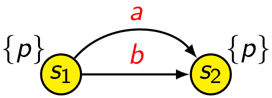

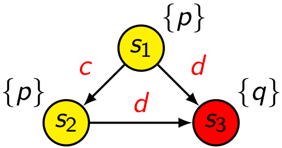

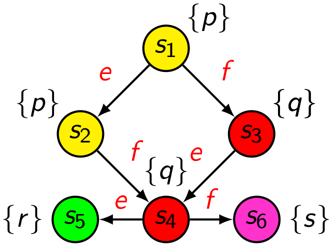

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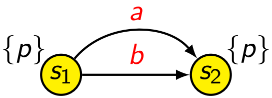

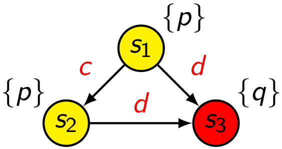

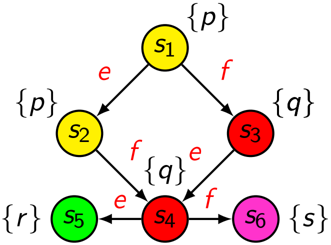

POR's notion of independence is stronger than necessary.

Strengthening of confluence

We can change confluence in the following way:

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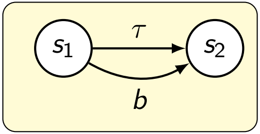

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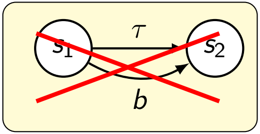

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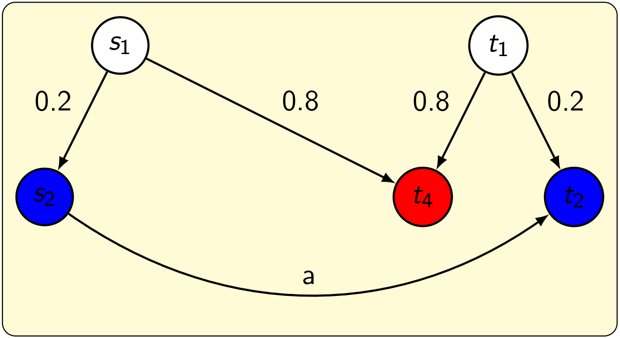

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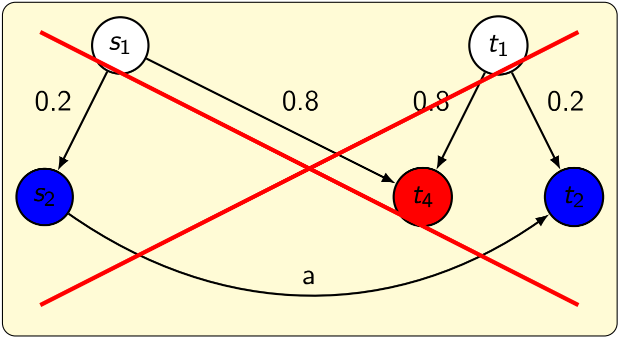

Strengthening of confluence

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In the non-probabilistic setting, the same statements hold: confluence is stronger than partial-order reduction, and the notions are equivalent for the strengthened variant of confluence.

Implications

State space generation using representatives:

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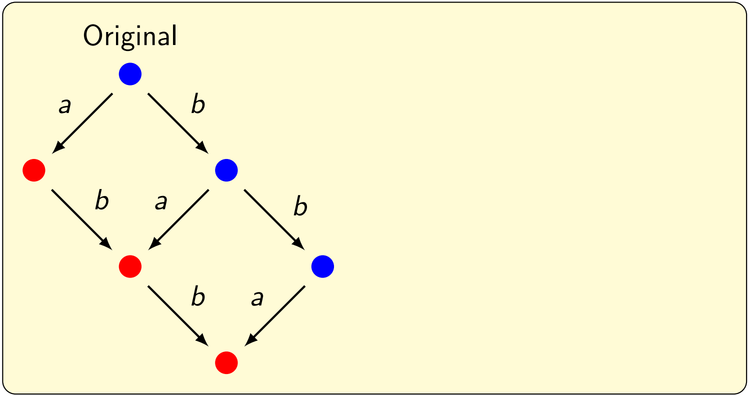

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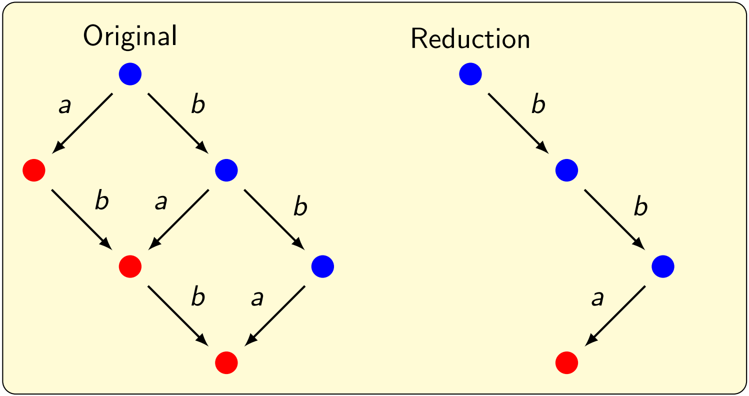

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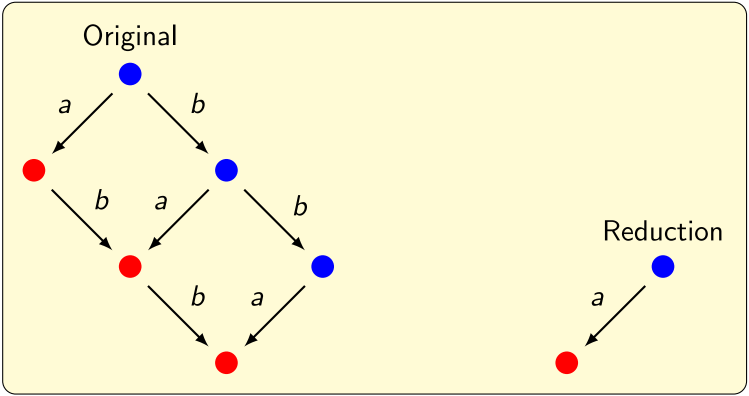

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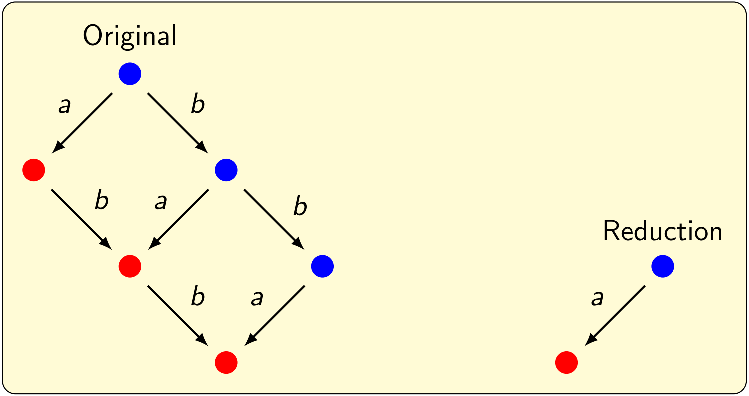

- Representative in bottom strongly connected component
- Additional reduction of states and transitions
- No need for the cycle condition anymore!

Conclusions

What to take home from this...

- We adapted the existing notion of confluence reduction to work in a state-based setting with MDPs.
- We proved that every ample set can be mimicked by a confluent set, but the the converse doesn't always hold.
- We showed how to make ample set reduction and confluence reduction equivalent
- We demonstrated one implication of our results, applying a technique from confluence reduction to POR
- The results are independent of specific heuristics, and also hold non-probabilistically

Questions

Questions?

A paper, containing all details and proofs, can be found at

`http://wwwhome.cs.utwente.nl/~timmer/research.php`