UNIVERSITY OF TWENTE.

Formal Methods & Tools.

Confluence Reduction for Probabilistic Systems

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September 4, 2010

Joint work with
Mariëlle Stoelinga and Jaco van de Pol

YR-CONCUR 2010

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The context: probabilistic model checking

Probabilistic model checking:

- Verifying quantitative properties,
- Using a probabilistic model (e.g., a probabilistic automaton)

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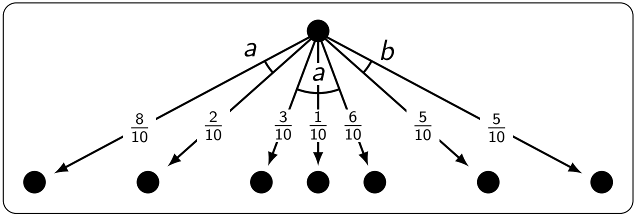

- Non-deterministically choose one of the three transitions
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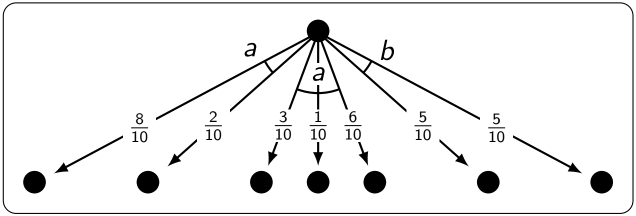

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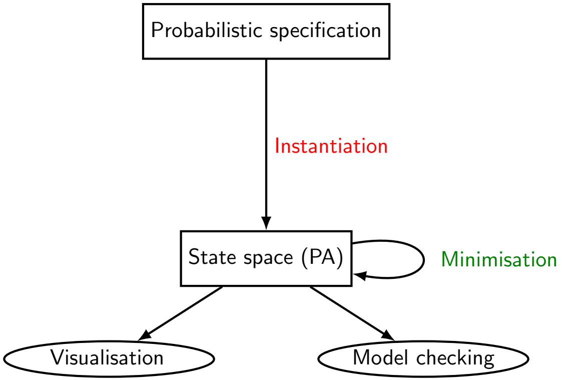
Limitations of previous approaches:

- Susceptible to the state space explosion problem
- Restricted treatment of data

Overview of our approach



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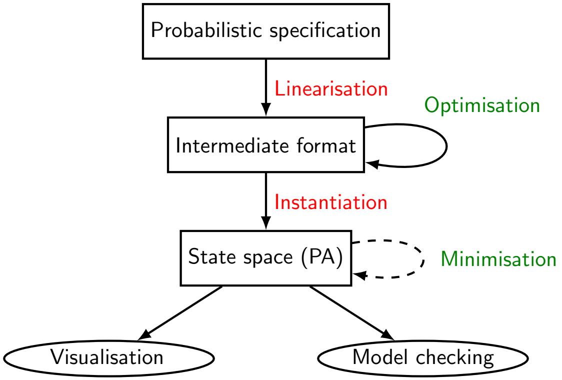

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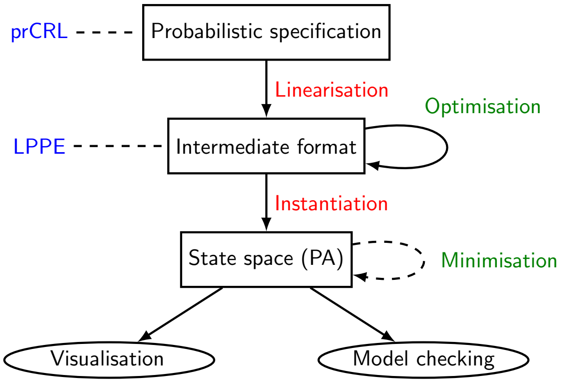

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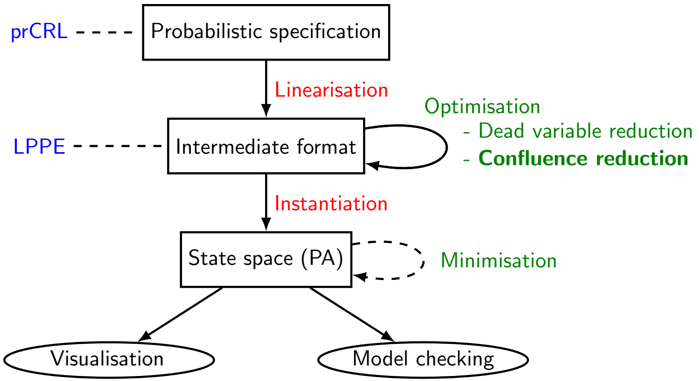
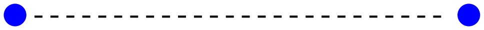


Branching probabilistic bisimulation

Notion of equivalence: branching probabilistic bisimulation



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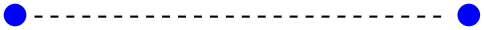

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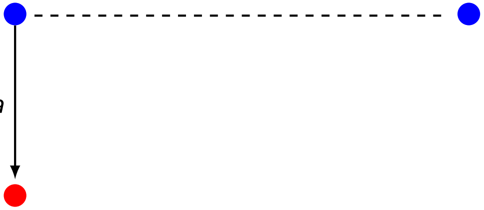

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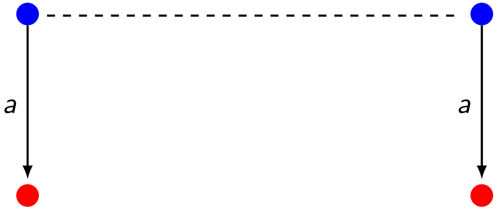

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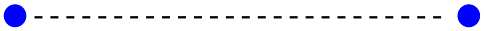

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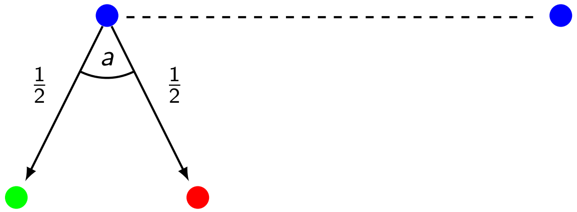

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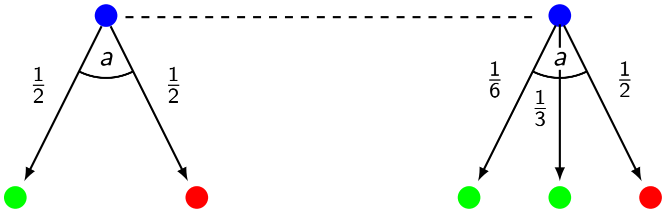

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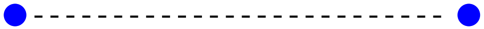

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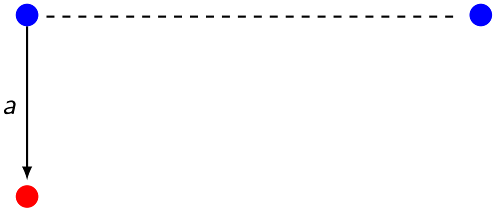

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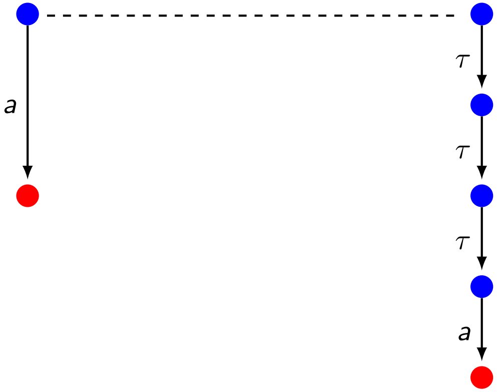

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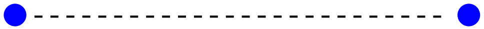

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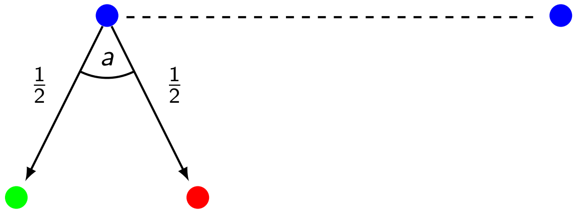

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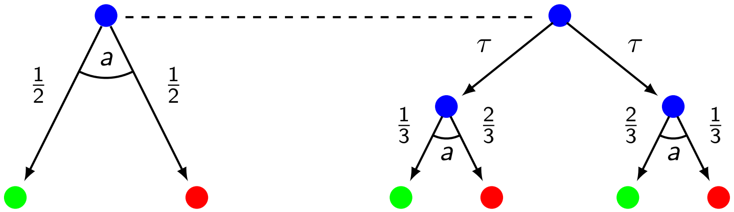

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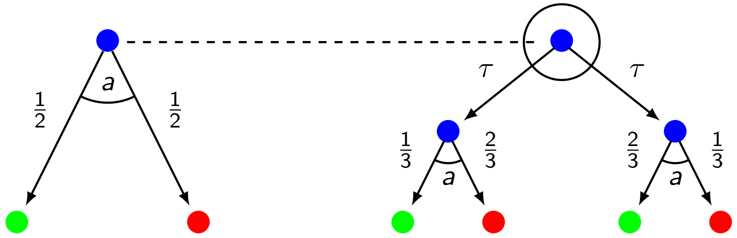

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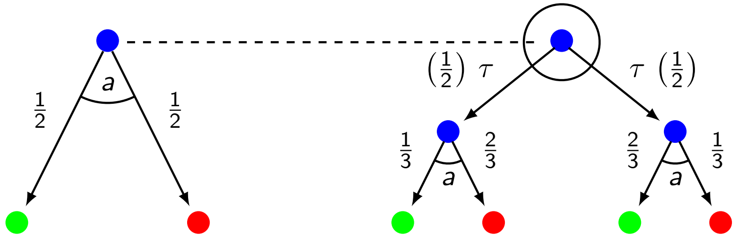

Probability of green: $\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{2}$

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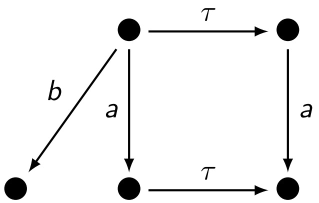
Branching bisimulation preservation by τ-steps

Unobservable τ-steps might disable behaviour...



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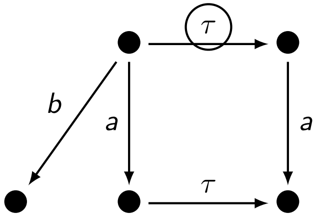

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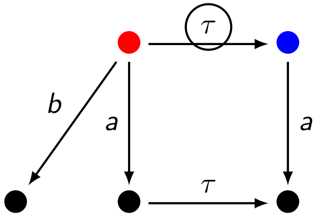

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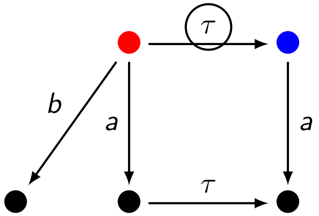

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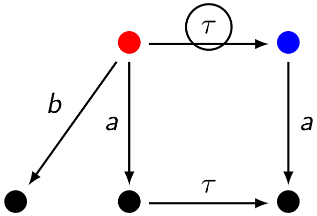

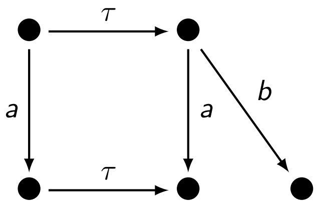

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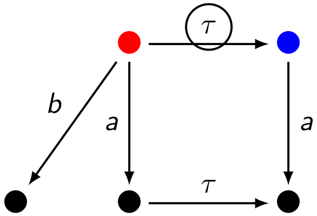

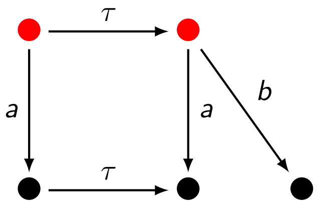

Confluence: an introductory example

$$\begin{tikzcd}\bullet \arrow[r, "\mathsf{getPen}"] & \bullet \arrow[r, "\mathsf{writeName}"] & \bullet\end{tikzcd}$$

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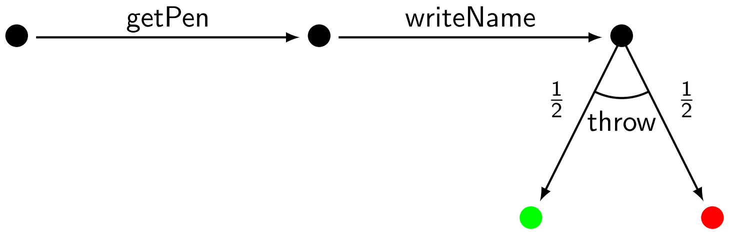

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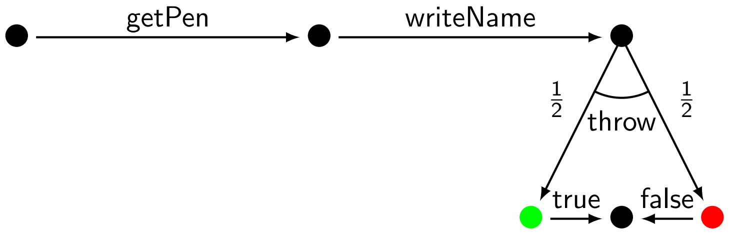

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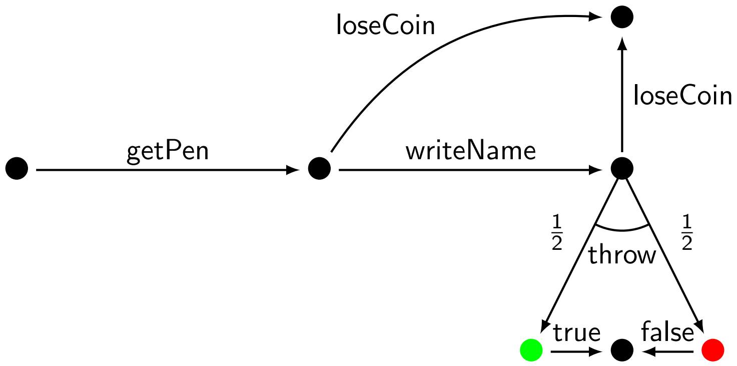

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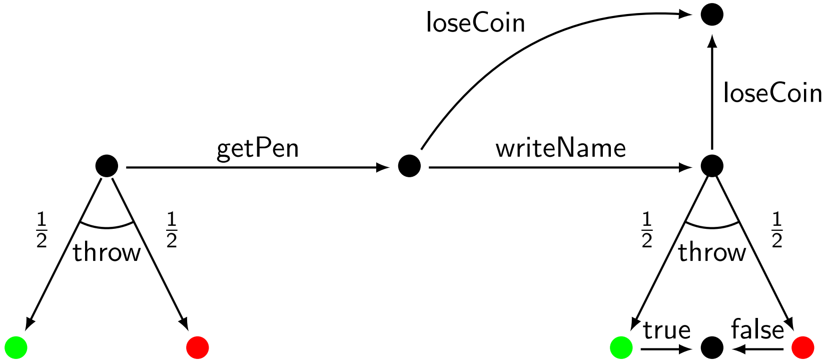

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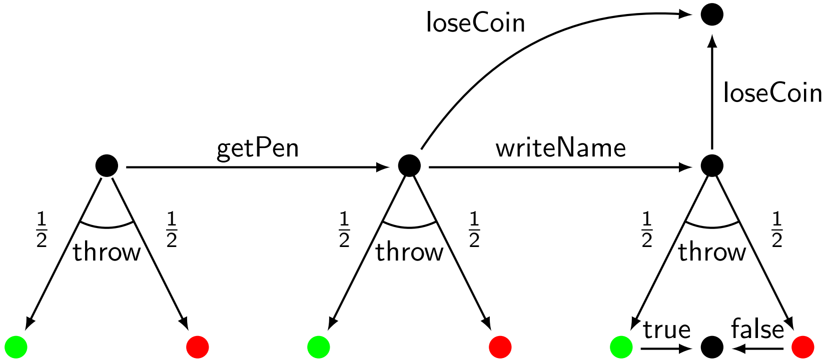

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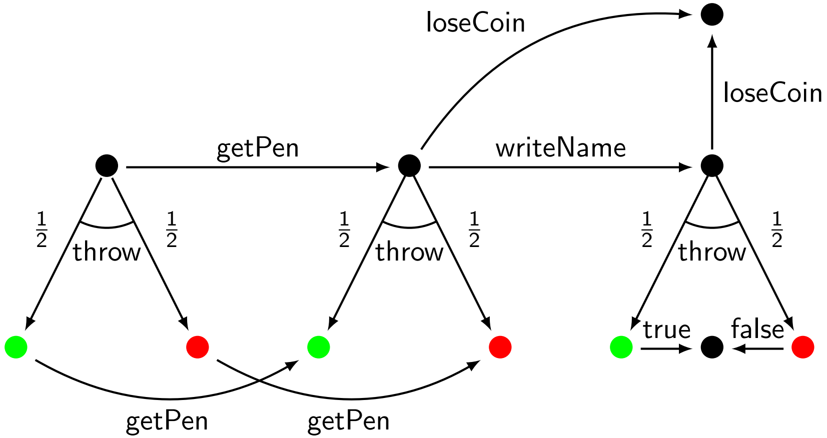

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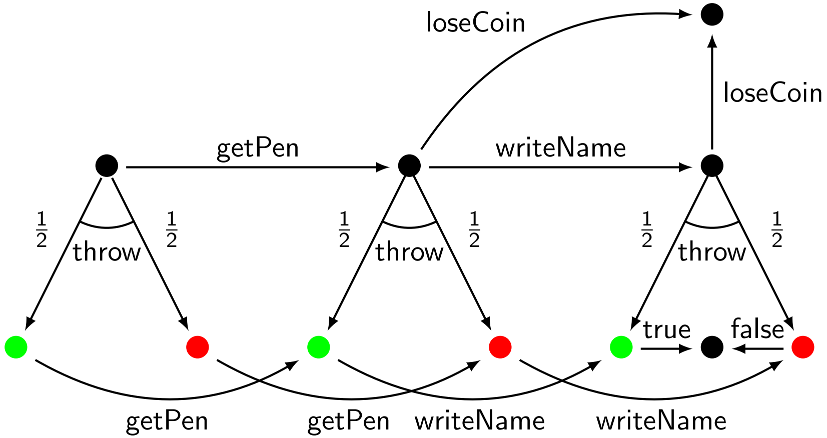

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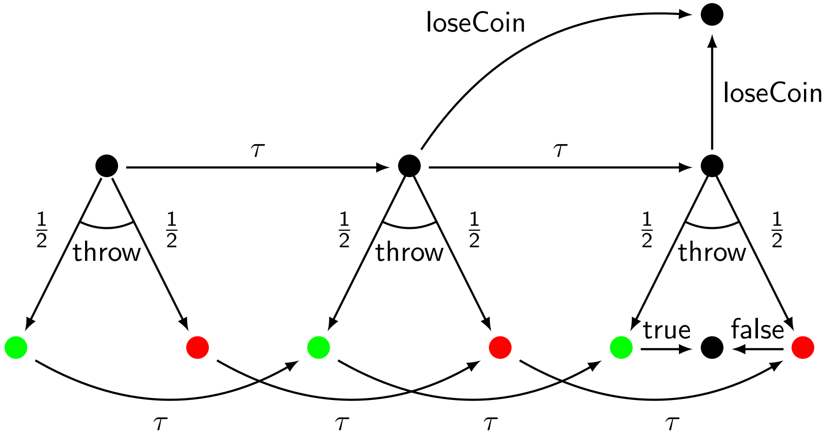

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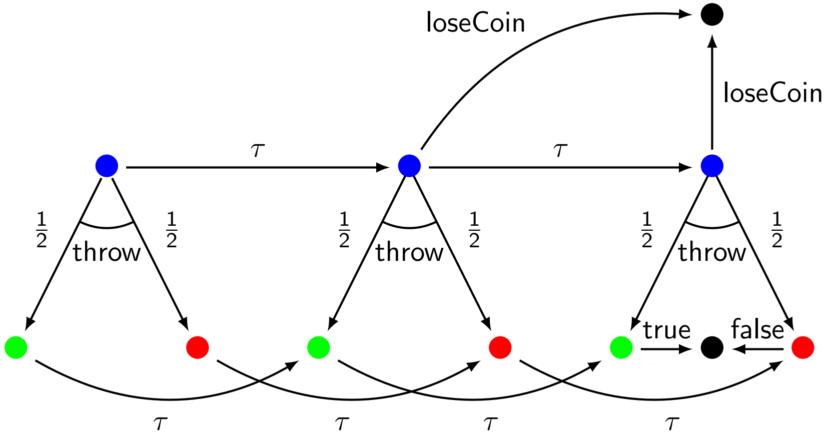

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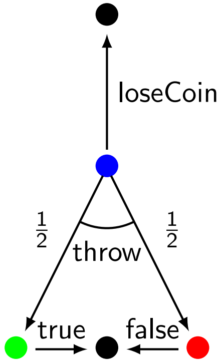

Confluence: non-probabilistic versus probabilistic

Three notions of confluence:

- weak confluence
- confluence
- strong confluence

Confluence: non-probabilistic versus probabilistic

Three notions of confluence:

• weak confluence		• weak probabilistic confluence
• confluence	$\Rightarrow$	• probabilistic confluence
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$$\begin{tikzcd}
\bullet \arrow[r, "\tau_{\mathrm{c}}"] \arrow[d, "a"'] & \bullet \arrow[d, dashed, "a"] \\
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\end{tikzcd}$$

Strong confluence

Confluence: non-probabilistic versus probabilistic

Three notions of confluence:

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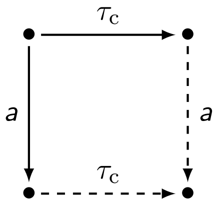
confluence	$\Rightarrow$	probabilistic confluence
strong confluence		strong probabilistic confluence



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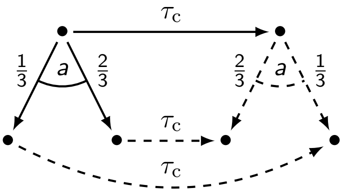
Strong confluence



Strong probabilistic confluence

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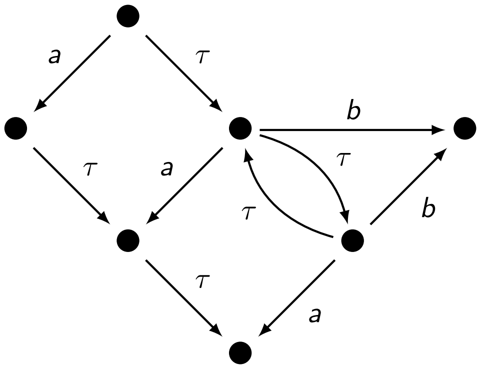
State space reduction using confluence



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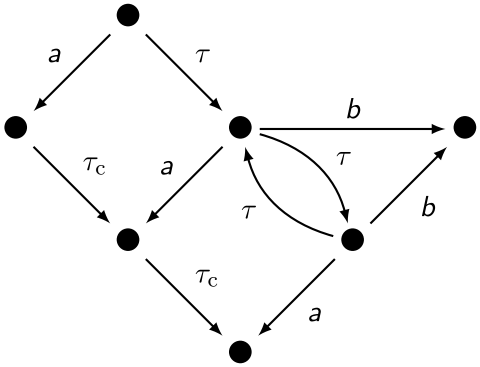

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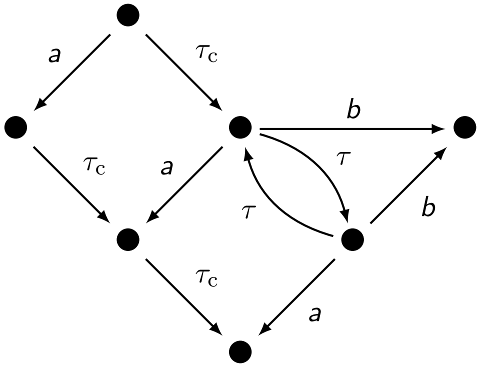

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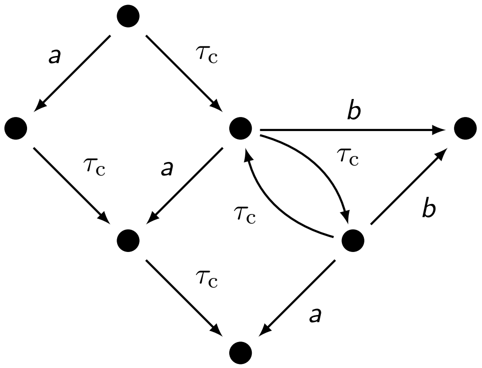

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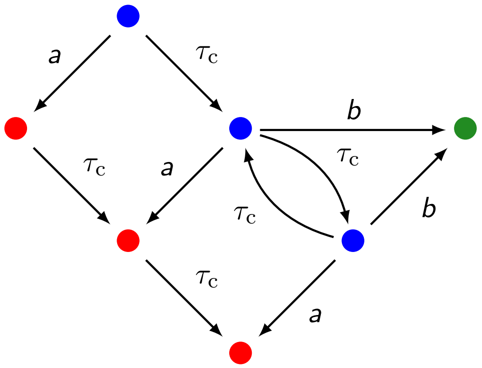

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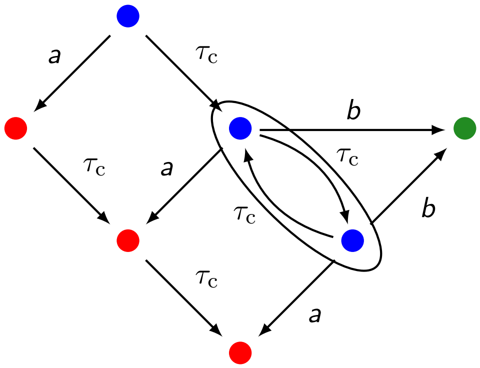

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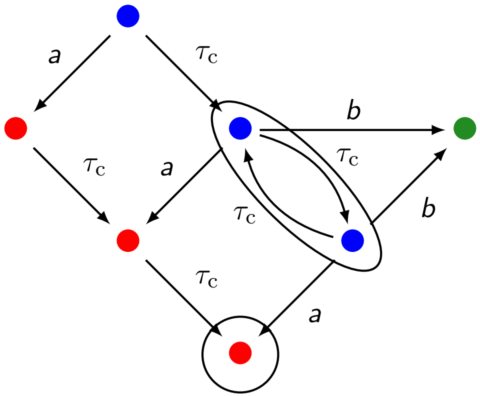

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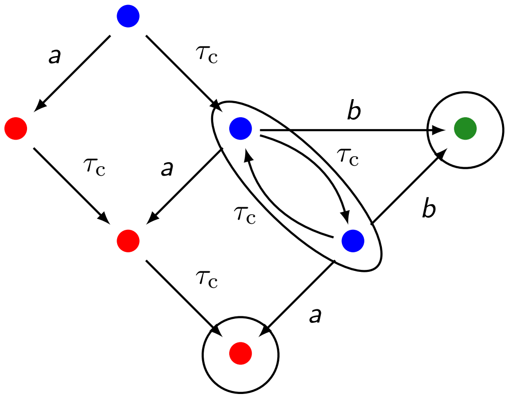

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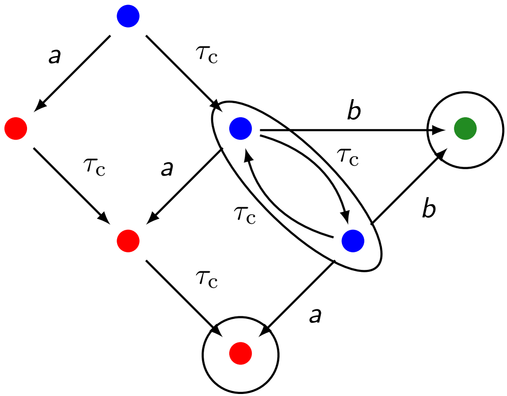

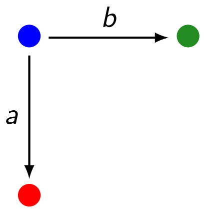

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Detecting confluence: LPPEs

We detect confluence symbolically using LPPEs:

$$
\begin{aligned}
X(\vec{g} : \vec{G}) = & \sum_{\vec{d_1}:\vec{D_1}} c_1 \Rightarrow a_1 \sum\!\!\!\!\bullet_{\vec{e_1}:\vec{E_1}} f_1 : X(\vec{n_1}) \\
& \dots \\
+ & \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k \sum\!\!\!\!\bullet_{\vec{e_k}:\vec{E_k}} f_k : X(\vec{n_k})
\end{aligned}
$$

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Example of an LPPE

$$\begin{aligned} &X(\mathrm{pc} : \{1..2\}, \mathrm{active} : \mathrm{Bool}) = \\ &\quad \sum_{n:\{1,2,3\}} \mathrm{pc} = 1 \Rightarrow \mathrm{output}(n) \sum\!\!\!\bullet_{\mathrm{b:Bool}} \tfrac{1}{2} : X(2, \mathrm{b}) \\ &+ \quad \mathrm{pc} = 2 \wedge \mathrm{active} \Rightarrow \mathrm{beep} \cdot X(1, \mathrm{active}) \end{aligned}$$

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$$\cdots$$
$$+ \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k \sum\!\!\!\bullet_{\vec{e_k}:\vec{E_k}} f_k : X(\vec{n_k})$$

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Detecting confluence: LPPEs

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How to know whether a summand is confluent?

Detecting confluence: LPPEs

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How to know whether a summand is confluent?

- Its action should be τ

Detecting confluence: LPPEs

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\end{aligned}
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How to know whether a summand is confluent?

- Its action should be τ
- Its next state should be chosen nonprobabilistically

Detecting confluence: LPPEs

Example of an LPPE

$$
\begin{aligned}
& X(\mathsf{pc} : \{1..2\}, \mathsf{active} : \mathsf{Bool}) = \\
& \qquad \sum_{n:\{1,2,3\}} \mathsf{pc} = 1 \qquad \Rightarrow \mathsf{output}(n) \sum\!\!\bullet_{\mathsf{b:Bool}} \tfrac{1}{2} : X(2, \mathsf{b}) \\
& + \qquad\qquad \mathsf{pc} = 2 \wedge \mathsf{active} \Rightarrow \tau \cdot X(1, \mathsf{active})
\end{aligned}
$$

How to know whether a summand is confluent?

- Its action should be τ
- Its next state should be chosen nonprobabilistically
- It should commute with all the other summands

Detecting confluence: LPPEs

Example of an LPPE

$$
\begin{aligned}
&X(\mathrm{pc} : \{1..2\}, \mathrm{active} : \mathrm{Bool}) = \\
&\quad \sum_{n:\{1,2,3\}} \mathrm{pc} = 1 \Rightarrow \mathrm{output}(n) \sum_{\mathrm{b:Bool}}\!\!\bullet\; \tfrac{1}{2} : X(2, \mathrm{b}) \\
&+ \quad \mathrm{pc} = 2 \wedge \mathrm{active} \Rightarrow \tau \cdot X(1, \mathrm{active})
\end{aligned}
$$

How to know whether a summand is confluent?

- Its action should be τ
- Its next state should be chosen nonprobabilistically
- It should commute with all the other summands
    - Never enabled at the same time
    - Not touching the same variables

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Case study: leader election protocol

	Original		Reduced		Running time	
Specification	States	Trans.	States	Trans.	Before	After
`leader`	3763	6158	1399	1922	1.86 sec	0.72 sec
`leaderReduced`	1693	2438	589	722	0.90 sec	0.44 sec
`leader-2-2`	67	94	27	32	0.04 sec	0.65 sec
`leader-2-6`	535	710	199	212	0.36 sec	0.81 sec
`leader-2-36`	18325	23690	6589	6662	516.23 sec	43.11 sec
`leader-3-2`	1018	1815	376	561	1.61 sec	3.81 sec
`leader-3-6`	21664	36519	7936	10233	221.22 sec	44.92 sec

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-60% -70%

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Conclusions

Related work

- Confluence reduction for non-probabilistic systems
- Partial-order reduction for probabilistic systems

Conclusions

Related work

- Confluence reduction for non-probabilistic systems
- Partial-order reduction for probabilistic systems

Conclusions / results

- We developed three new notions of confluence for probabilistic automata that preserve branching probabilistic bisimulation;
- We showed how confluence can be used for state space reduction;
- We discussed how to detect confluence based on a probabilistic process algebra;
- We illustrated the power of our method using a case study.

Questions

Questions?