UNIVERSITY OF TWENTE.

Formal Methods & Tools.

A linear process algebraic format for probabilistic systems

Mark Timmer

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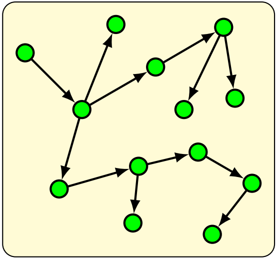
Joint work with Joost-Pieter Katoen, Jaco van de Pol, and Mariëlle Stoelinga

Contents

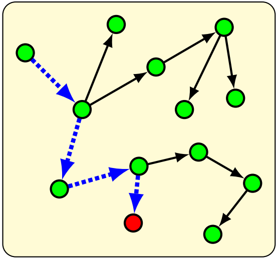
Introduction – Probabilistic Model Checking

Probabilistic model checking:

> *Verifying properties of a system containing probabilistic choices by constructing a model and ranging over its entire state space.*



Pass

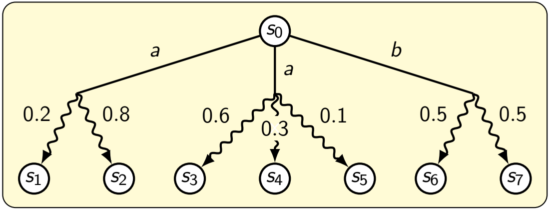


Fail

Introduction – Probabilistic Model Checking

Probabilistic model checking:

> *Verifying properties of a system containing probabilistic choices by constructing a model and ranging over its entire state space.*



Introduction – Probabilistic Model Checking

Probabilistic model checking:

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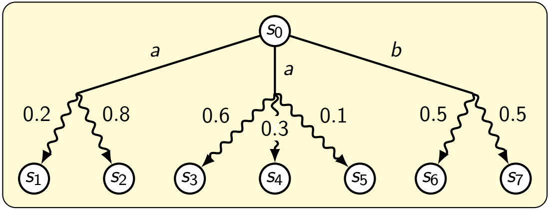


Disadvantages:

- Susceptible to the state space explosion problem
- Restricted treatment of data

Introduction – The non-probabilistic setting

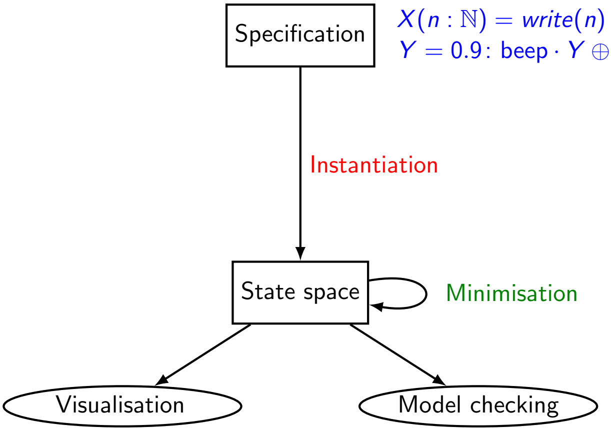
Higher-order languages (often process algebras) are used to simplify state space specification and incorporate data.

Introduction – The non-probabilistic setting

Higher-order languages (often process algebras) are used to simplify state space specification and incorporate data.

$X(n : \mathbb{N}) = \mathit{write}(n) \cdot X(n+1)$

$Y = 0.9 : \text{beep} \cdot Y \oplus 0.1 : \text{crash}$



Introduction – The non-probabilistic setting

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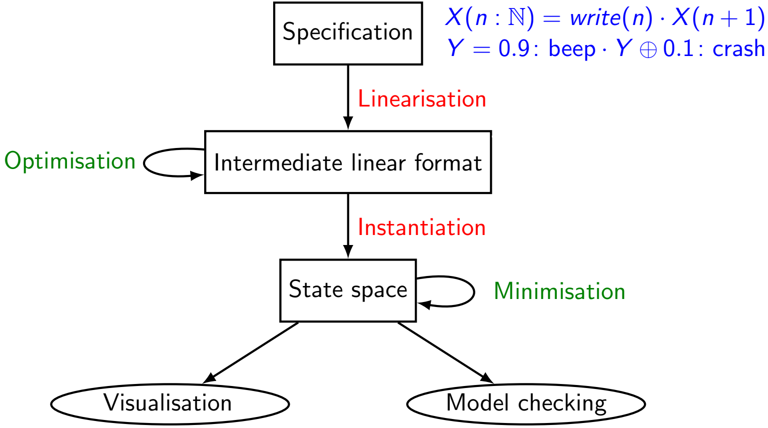

Introductions – Contributions

Our contributions:

1. A process algebra called prCRL, incorporating both data and probability
2. A linear format for this algebra: the LPPE
3. An algorithm and implementation for linearisation, including a thorough mathematical correctness proof

A process algebra with data and probability: prCRL

The grammar of prCRL process terms

Process terms in prCRL are obtained by the following grammar:

$$p ::= Y(\vec{t}) \mid c \Rightarrow p \mid p + p \mid \sum_{x:D} p \mid a(\vec{t}) \sum_{x:D}\!\!\bullet\; f : p$$

- c is a condition (boolean expression)
- a is an atomic action
- f is a real-valued expression yielding values in $[0, 1]$
- $\vec{t}$ is a vector of expressions

A process algebra with data and probability: prCRL

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Process equations and processes

A process equation is something of the form $X(\vec{g} : \vec{G}) = p$.

Some examples

Sending an arbitrary natural number

$$X = \tau \sum_{n:\mathbb{N}^{>0}}\!\!\!\!\bullet\; \frac{1}{2^n} : (\text{send}(n) \cdot \sum_{j:\{*\}}\!\!\!\!\bullet\; 1.0 : X)$$

Some examples

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Sending ping messages until system crash

$$B = \mathrm{ping} \sum_{i : \{1,2\}}^{\bullet} (\text{if } i = 1 \text{ then } 0.1 \text{ else } 0.9) :$$

$$((i = 1 \Rightarrow \mathrm{crash} \cdot B) + (i \neq 1 \Rightarrow B))$$

Some examples

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Sending ping messages until system crash

$$B = \text{ping}(0.1 : \text{crash} \cdot B \oplus 0.9 : B)$$

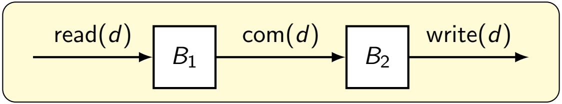
Linear process equations

In the non-probabilistic setting, LPEs are given by

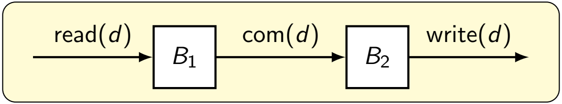
$$\begin{aligned} X(\vec{g} : \vec{G}) = & \sum_{\vec{d_1}:\vec{D_1}} c_1 \Rightarrow a_1(b_1) \cdot X(n_1) \\ & \dots \\ & + \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k(b_k) \cdot X(n_k) \end{aligned}$$

- $\vec{G}$ is a type for state vectors
- $\vec{D}_i$ a type for local variable vectors for summand i
- c_i is the enabling condition of summand i
- a_i is an atomic action, with action-parameter vector b_i
- n_i is the next-state vector of summand i.

Linear process equations – An example

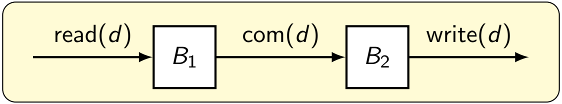


Linear process equations – An example



$$B_1 = \sum_{d:D} \text{read}(d) \cdot \text{com}(d) \cdot B_1 \qquad B_2 = \sum_{d:D} \overline{\text{com}}(d) \cdot \text{write}(d) \cdot B_2$$

Linear process equations – An example



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$$\begin{aligned}
&X(a : \{1,2\}, b : \{1,2\}, x : D, y : D) = \\
&\quad \textstyle\sum_{d:D} \quad a = 1 && \Rightarrow \mathrm{read}(d) \cdot X(2, b, d, y) && (1)\\
&+ \quad a = 2 \wedge b = 1 && \Rightarrow \mathrm{com}(x) \cdot X(1, 2, x, x) && (2)\\
&+ \quad b = 2 && \Rightarrow \mathrm{write}(y) \cdot X(a, 1, x, y) && (3)
\end{aligned}$$

A linear format for prCRL: the LPPE

In the probabilistic setting, LPPEs are given by

$$X(\vec{g} : \vec{G}) = \sum_{\vec{d_1}:\vec{D_1}} c_1 \Rightarrow a_1(b_1) \sum\!\!\!\bullet_{\vec{e_1}:\vec{E_1}} f_1 : X(n_1)$$

$$\dots$$

$$+ \sum_{\vec{d_k}:\vec{D_k}} c_k \Rightarrow a_k(b_k) \sum\!\!\!\bullet_{\vec{e_k}:\vec{E_k}} f_k : X(n_k)$$

An example

$$X(\mathrm{pc} : \{1,2\}, d : D) = \mathrm{pc} = 1 \Rightarrow \tau \sum\!\!\!\bullet_{e:D} \frac{1}{|D|} : X(2, e)$$

$$+ \mathrm{pc} = 2 \Rightarrow \mathit{send}(d) \cdot X(1, d))$$

A linear format for prCRL: the LPPE

An example

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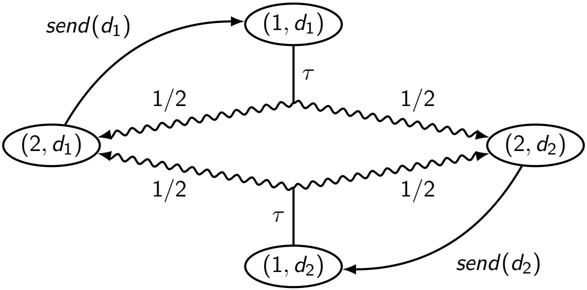

Linearisation

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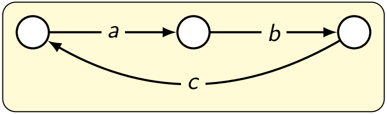

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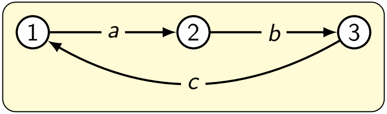

Linearisation

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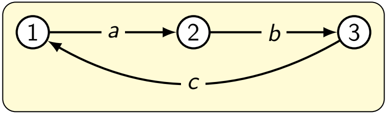
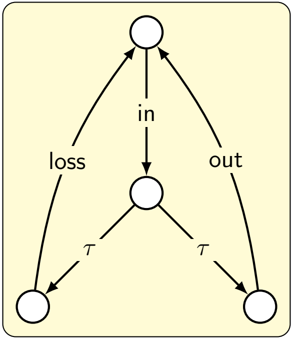


$$\begin{aligned} Y(pc\colon \{1,2,3\}) = \\ pc = 1 \Rightarrow a \cdot Y(2) \\ + pc = 2 \Rightarrow b \cdot Y(3) \\ + pc = 3 \Rightarrow c \cdot Y(1) \end{aligned}$$

Linearisation

$$X = \sum_{d:D} \mathsf{in}(d) \cdot (\tau \cdot \mathsf{loss} \cdot X + \tau \cdot \mathsf{out}(d) \cdot X)$$



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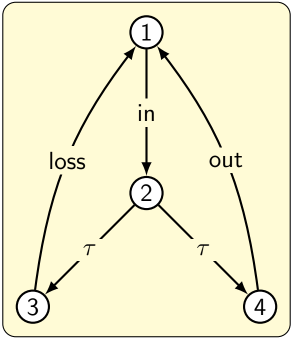
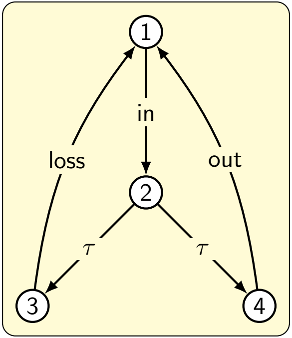


Linearisation

$$X = \sum_{d\colon D} \text{in}(d) \cdot (\tau \cdot \text{loss} \cdot X + \tau \cdot \text{out}(d) \cdot X)$$



$$\begin{aligned} Y(pc\colon \{1,2,3,4\}, x\colon D) = \\ & \textstyle\sum_{d:D} \quad pc = 1 \Rightarrow \text{in}(d) \cdot Y(2,d) \\ + & \quad pc = 2 \Rightarrow \tau \cdot Y(3,x) \\ + & \quad pc = 2 \Rightarrow \tau \cdot Y(4,x) \\ + & \quad pc = 3 \Rightarrow \text{loss} \cdot Y(1,x) \\ + & \quad pc = 4 \Rightarrow \text{out}(x) \cdot Y(1,x) \end{aligned}$$

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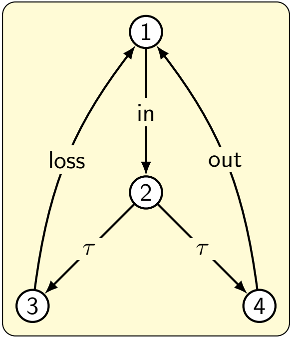

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\textstyle\sum_{d:D} \quad & pc = 1 \Rightarrow \mathrm{in}(d) \cdot Y(2, d) \\
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+ \quad & pc = 3 \Rightarrow \mathrm{loss} \cdot Y(1, x) \\
+ \quad & pc = 4 \Rightarrow \mathrm{out}(x) \cdot Y(1, x)
\end{aligned}
$$

Initial process: $Y(1, d_1)$.

Linearisation

$$X(d : D) = \sum_{e:D} a(d + e) \sum_{f:D} \bullet \frac{1}{|D|} : \left(c(e) \cdot c(f) \cdot X(5) + c(e + f) \cdot X(5) \right)$$

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2 $X_1(d : D, e : D, f : D) = \sum_{e:D} a(d+e) \sum\!\!\!\!\bullet_{f:D} \frac{1}{|D|} \cdot X_2(d, e, f)$

Linearisation

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$$X(d : D) = \sum_{e:D} a(d+e) \sum_{f:D} \bullet \frac{1}{|D|} : \left(c(e) \cdot c(f) \cdot X(5) + c(e+f) \cdot X(5)\right)$$

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Linearisation

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$\sum_{e:D} a(d+e) \sum_{\bullet f:D} \frac{1}{|D|} : (c(e) \cdot c(f) \cdot X(5) + c(e+f) \cdot X(5))$

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2 $X_1(d : D, e : D, f : D) = \sum_{e:D} a(d+e) \sum_{\bullet f:D} \frac{1}{|D|} \cdot X_2(d, e, f)$
$X_2(d : D, e : D, f : D) = c(e) \cdot c(f) \cdot X(5) + c(e+f) \cdot X(5)$

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3 $X_1(d : D, e : D, f : D) = \sum_{e:D} a(d+e) \sum_{\bullet f:D} \frac{1}{|D|} \cdot X_2(d, e, f)$
$X_2(d : D, e : D, f : D) = c(e) \cdot X_3(d, e, f) + c(e+f) \cdot X_1(5, e, f)$
$X_3(d : D, e : D, f : D) = c(f) \cdot X(5)$

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4 $X_1(d : D, e : D, f : D) = \sum_{e:D} a(d+e) \sum_{\bullet f:D} \frac{1}{|D|} \cdot X_2(d, e, f)$
$X_2(d : D, e : D, f : D) = c(e) \cdot X_3(d, e, f) + c(e+f) \cdot X_1(5, e, f)$
$X_3(d : D, e : D, f : D) = c(f) \cdot X_1(5, e, f)$

Linearisation

$$X(d : D) = \sum_{e:D} a(d + e) \sum_{f:D}^{\bullet} \frac{1}{|D|} : \left(c(e) \cdot c(f) \cdot X(5) + c(e + f) \cdot X(5) \right)$$

$$X_1(d : D, e : D, f : D) = \textstyle\sum_{e:D} a(d + e) \sum^{\bullet}_{f:D} \frac{1}{|D|} \cdot X_2(d, e, f)$$
$$X_2(d : D, e : D, f : D) = c(e) \cdot X_3(d, e, f) + c(e + f) \cdot X_1(5, e, f)$$
$$X_3(d : D, e : D, f : D) = c(f) \cdot X_1(5, e, f)$$

Linearisation

$$X(d:D) = \sum_{e:D} a(d+e) \sum_{f:D}^{\bullet} \tfrac{1}{|D|} : \left(c(e) \cdot c(f) \cdot X(5) + c(e+f) \cdot X(5) \right)$$

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$$X_2(d:D, e:D, f:D) = c(e) \cdot X_3(d,e,f) + c(e+f) \cdot X_1(5,e,f)$$
$$X_3(d:D, e:D, f:D) = c(f) \cdot X_1(5,e,f)$$

$$\begin{aligned} X(\mathrm{pc}:\{1,2,3\}, d:D, e:D, f:D) = \\ \mathrm{pc} = 1 \Rightarrow \sum_{e:D} a(d+e) \sum_{f:D}^{\bullet} \frac{1}{|D|} \cdot X(2,d,e,f) \\ + \mathrm{pc} = 2 \Rightarrow c(e) \cdot X(3,d,e,f) \\ + \mathrm{pc} = 2 \Rightarrow c(e+f) \cdot X(1,5,e,f) \\ + \mathrm{pc} = 3 \Rightarrow c(f) \cdot X(1,5,e,f) \end{aligned}$$

Extended prCRL

The grammar of extended prCRL process terms

Process terms in extended prCRL are obtained by:

$$q ::= p \mid q \,||\, q \mid \partial_E(q) \mid \tau_H(q) \mid \rho_R(q)$$

Linearisation can be done compositionally: transform every subsystem to LPPE, and put them in parallel.

Conclusions and Future Work

Conclusions / Results

- We developed the process algebra prCRL, incorporating both data and probability.
- We defined a linear format for prCRL, the LPPE, providing the starting point for effective symbolic optimisations and easy state space generation.
- We provided a linearisation algorithm to transform prCRL specifications to LPPEs, proved it correct, and implemented it.

Future work

Applying existing optimisation techniques to LPPEs

- constant elimination
- liveness analysis
- confluence reduction