State Space Reduction of Linear Processes using Control Flow Reconstruction

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University of Twente

FMT Lunchmeeting
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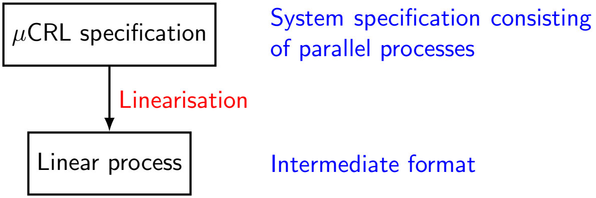

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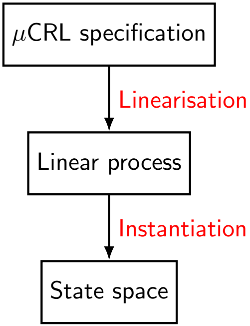
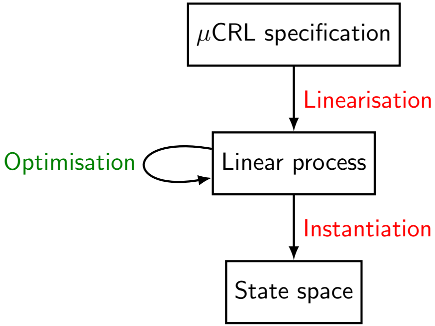


System specification consisting of parallel processes

Intermediate format

Very big graph

The μCRL toolset



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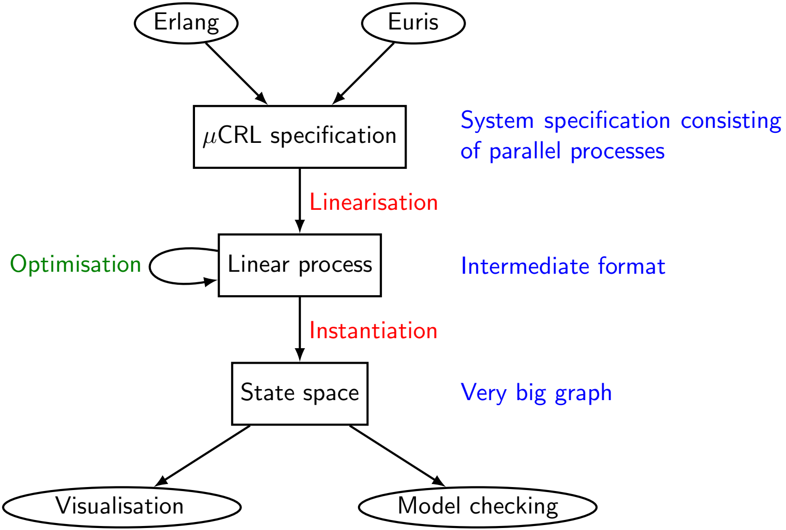
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Linearisation
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State space
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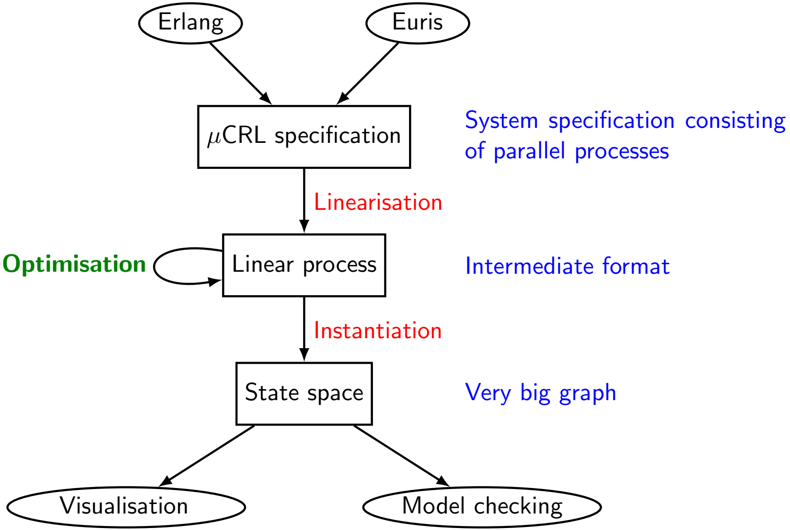

The linear process equation

The basic structure of an LPE

$$
\begin{aligned}
X(d\colon D) = & \sum_{e_1 : E_1} c_1(d, e_1) \Rightarrow a_1(d, e_1) \cdot X(g_1(d, e_1)) \\
+ & \quad \ldots \\
+ & \sum_{e_n : E_n} c_n(d, e_n) \Rightarrow a_n(d, e_n) \cdot X(g_n(d, e_n))
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- d: a vector of state variables
- e_i: a vector of local variables for summand i
- c_i: the enabling condition for summand i
- a_i: the (parameterised) action for summand i (possibly τ)
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$$\boldsymbol{d} \xrightarrow{a(\boldsymbol{p})} \boldsymbol{d'} \Leftrightarrow \exists i . \exists \boldsymbol{e_i} . c_i(\boldsymbol{d}, \boldsymbol{e_i}) = \mathtt{true} \wedge a_i(\boldsymbol{d}, \boldsymbol{e_i}) = a(\boldsymbol{p}) \wedge g_i(\boldsymbol{d}, \boldsymbol{e_i}) = \boldsymbol{d'}$$

An example

$$X = \sum_{d\,:\,D} \mathrm{in}(d) \cdot (\tau \cdot \mathrm{loss} \cdot X + \tau \cdot \mathrm{out}(d) \cdot X)$$

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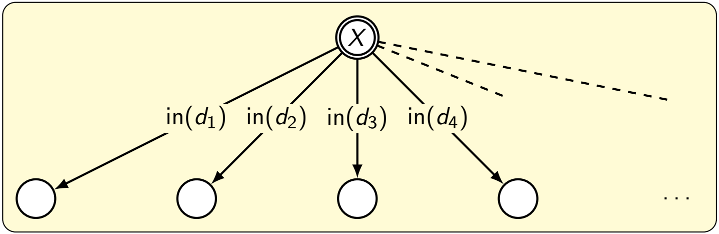

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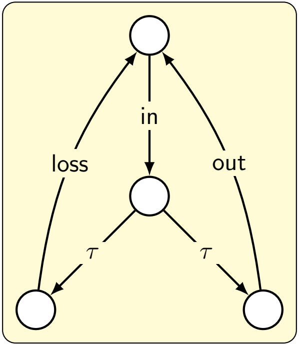

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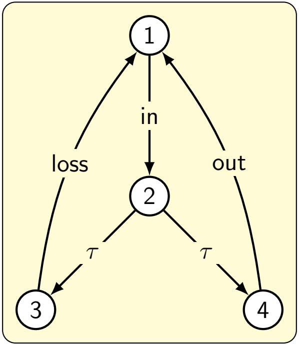

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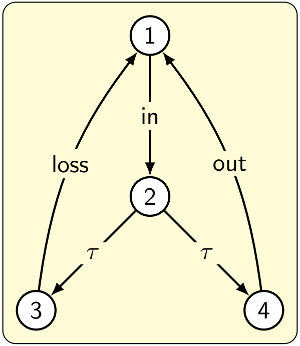

$$\begin{aligned}
& X(pc\colon \{1,2,3,4\}, x\colon D) = \\
& \quad \textstyle\sum_{d:\,D}\; pc = 1 \Rightarrow \mathsf{in}(d) \cdot X(2,d) \\
& + \quad pc = 2 \Rightarrow \tau \cdot X(3,x) \\
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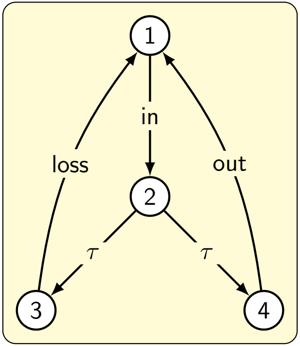

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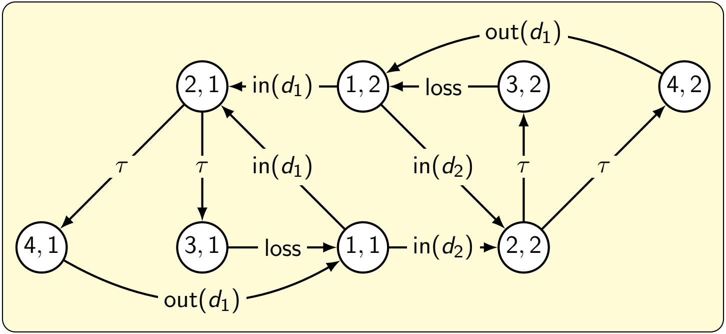
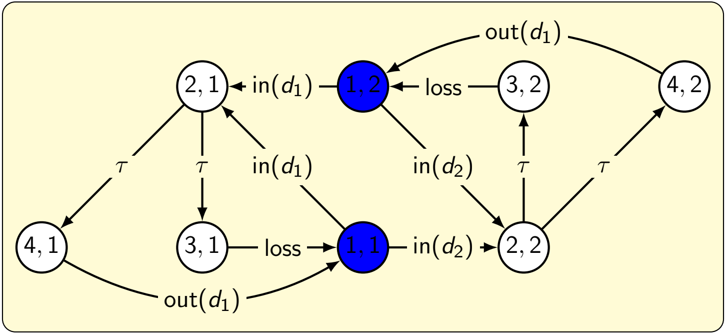
Initial process: $X(1, d_1)$.


out(d_1)
2, 1
in(d_1)
1, 2
loss
3, 2
4, 2
τ
τ
in(d_1)
in(d_2)
τ
τ
4, 1
3, 1
loss
1, 1
in(d_2)
2, 2
out(d_1)


out(d_1)
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loss
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4, 2
τ
τ
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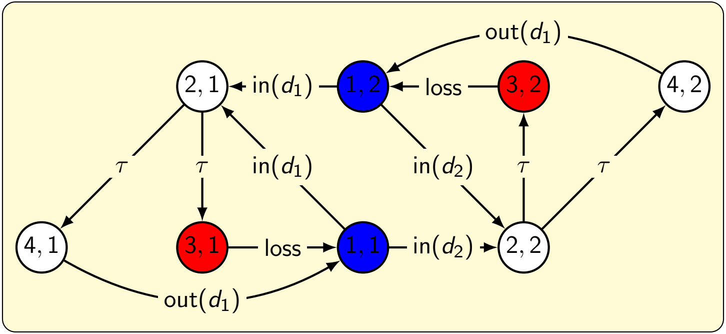
in(d_2)
τ
τ
4, 1
3, 1
loss
1, 1
in(d_2)
2, 2
out(d_1)


out(d_1)
2, 1
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1, 2
loss
3, 2
4, 2
τ
τ
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in(d_2)
τ
τ
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3, 1
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1, 1
in(d_2)
2, 2
out(d_1)

Problem: control flow is hidden in state parameters

Control Flow Reconstruction

Problem: control flow is hidden in state parameters

Solution:

1. Detect control flow parameters
2. Identify clusters of summands
3. Assign data parameters to clusters
4. Deduce when data parameters are relevant

Contents

Control flow parameters

Observation:
program counters (control flow parameters) are special.

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$$\boxed{B_1 = \sum_{d\,:\,D} read(d) \cdot w(d) \cdot B_1} \quad \boxed{B_2 = \sum_{d\,:\,D} r(d) \cdot write(d) \cdot B_2}$$

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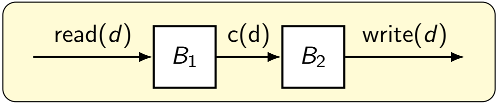

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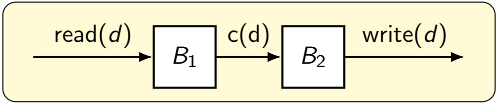

$$
\begin{array}{lll}
\multicolumn{3}{l}{X(a\colon \{1,2\}, b\colon \{1,2\}, x\colon D, y\colon D) =} \\
\sum_{d\colon D} & a = 1 & \Rightarrow \mathrm{read}(d) \cdot X(2, b, d, y) \\
+ & b = 2 & \Rightarrow \mathrm{write}(y) \cdot X(a, 1, x, y) \\
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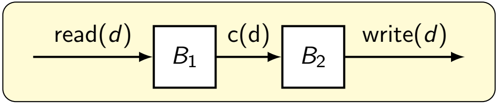

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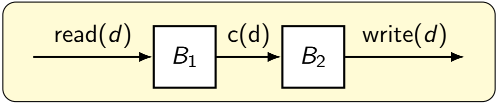
In every summand, each control flow parameter

- is either left unchanged, or
- has a clear transition from a source value to a destination value

Control flow parameters



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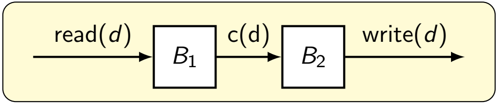

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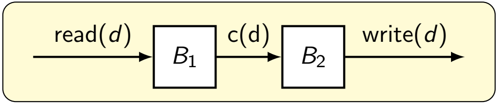

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+ \quad & a = {\color{red}2} \wedge b = {\color{red}1} && \Rightarrow c(x) \cdot X({\color{red}1}, {\color{red}2}, x, x)
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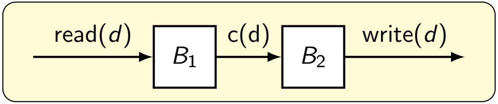

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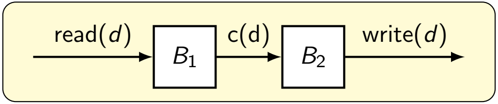

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In every summand, each control flow parameter

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Control Flow Graph

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\quad \sum_{d\colon D} & a = 1 & \Rightarrow \mathrm{read}(d) \cdot X(2, b, d, y) & & (1) \\
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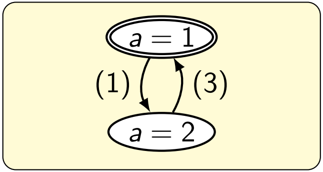

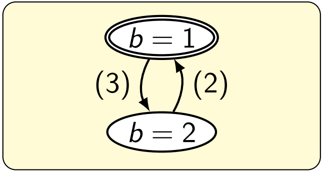

Contents

The *belongs to* relation

A data parameter k belongs to a CFP j if the cluster of j contains all summands that

- either change k, or
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\end{array}
$$

So, x belongs to a and y belongs to b.

Thus, relevance of x can be decided by the control flow of a.

Relevance

$R(k, j, s)$: parameter k is relevant when CFP j is in state s

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$R(k, j, s)$: parameter k is relevant when CFP j is in state $s \iff$

There is a summand that can be taken when $d_j = s$, that either

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So: $R(y, b, 2)$

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+ & b = 2 & \Rightarrow \mathrm{write}(y) \cdot X(a, 1, x, y) & (2) \\
+ & a = 2 \wedge b = 1 & \Rightarrow \tau \cdot X(1, 2, x, x) & (3)
\end{array}
$$

So: $R(y, b, 2)$ and $R(x, a, 2)$.

Relevance

$R(k,j,s)$: parameter k is relevant when CFP j is in state $s \iff$

There is a summand that can be taken when $d_j = s$, that either

- directly uses k for its condition or action, or
- indirectly uses k to determine the value of a parameter that is relevant after taking the summand

$$
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So: $R(y,b,2)$ and $R(x,a,2)$.

If $\neg R(k,j,s)$, then k is irrelevant when j is in state s

Contents

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Based on data flow analysis, irrelevant parameters can be changed.

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So, assuming initially $x = y = d_1$

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Example

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\end{array}
$$

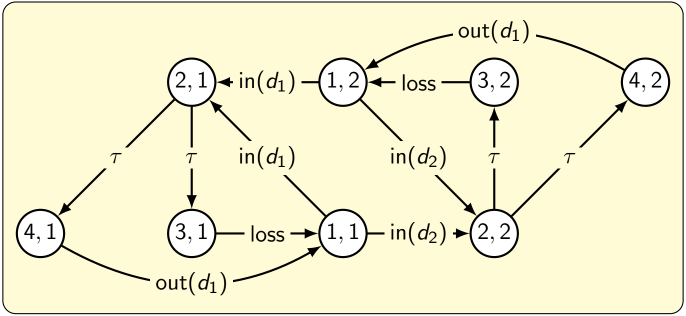
Now:

- $X(1, 2, x, y)$ are only reachable for $x = d_1$
- $X(2, 1, x, y)$ are only reachable for $y = d_1$

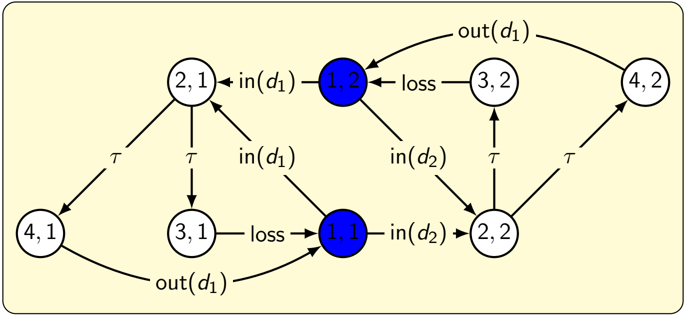
For $|D| = 5$, state space reduction of 60 to 36 states

For $|D| = n$, reduction of $2n^2 + 2n$ to $n^2 + 2n + 1$ states
(so a decrease of $n^2 - 1$ states)

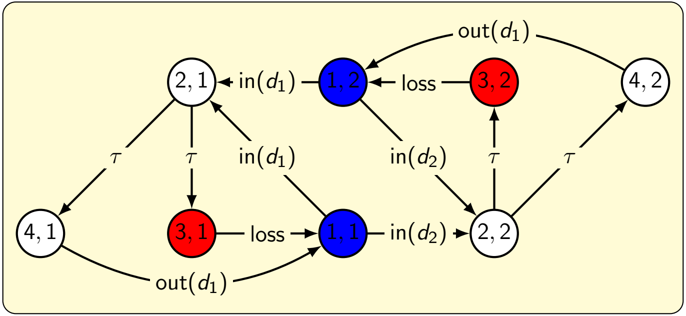
Example


out(d_1)
2, 1
in(d_1)
1, 2
loss
3, 2
4, 2
τ
τ
in(d_1)
in(d_2)
τ
τ
4, 1
3, 1
loss
1, 1
in(d_2)
2, 2
out(d_1)

Example

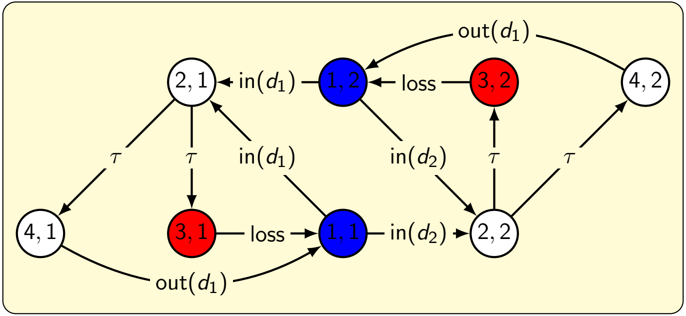

out(d_1)
2, 1
in(d_1)
1, 2
loss
3, 2
4, 2
τ
τ
in(d_1)
in(d_2)
τ
τ
4, 1
3, 1
loss
1, 1
in(d_2)
2, 2
out(d_1)

Example

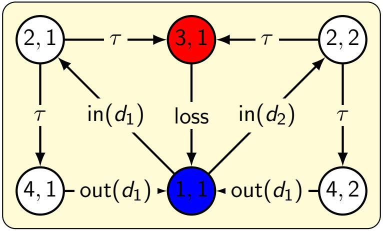


out(d_1)
2, 1
in(d_1)
1, 2
loss
3, 2
4, 2
τ
τ
in(d_1)
in(d_2)
τ
τ
4, 1
3, 1
loss
1, 1
in(d_2)
2, 2
out(d_1)

Example



out(d_1)
2, 1
in(d_1)
1, 2
loss
3, 2
4, 2
τ
τ
in(d_1)
in(d_2)
τ
τ
4, 1
3, 1
loss
1, 1
in(d_2)
2, 2
out(d_1)



2, 1
τ
3, 1
τ
2, 2
τ
in(d_1)
loss
in(d_2)
τ
4, 1
out(d_1)
1, 1
out(d_1)
4, 2

Correctness and effectiveness

Theorem: correctness

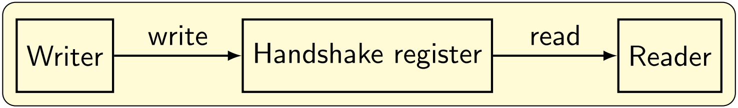
The transformed LPE is strongly bisimilar to the original

Theorem: effectiveness

The number of reachable states of the transformed LPE is at most as large as the number of reachable states in the original

Contents

A handshake register



- Recentness

  Any value read was at some point during the read action the last value written

- Sequentiality

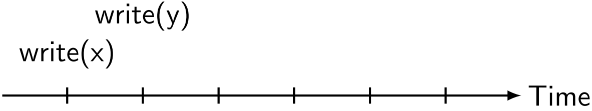
  The values of sequential reads occur in the same order as they were written

- Waitfree

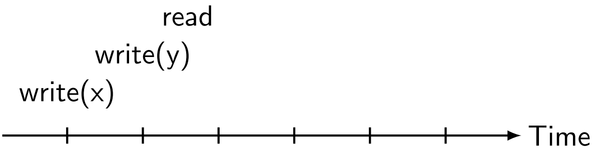
  Completion of reads/writes in a bounded number of steps

Building blocks for the handshake register

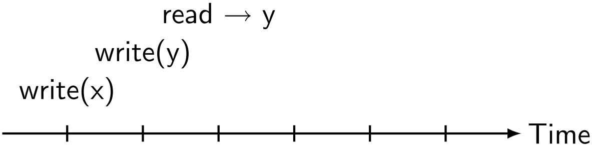
Building blocks for the handshake register



Building blocks for the handshake register



Building blocks for the handshake register



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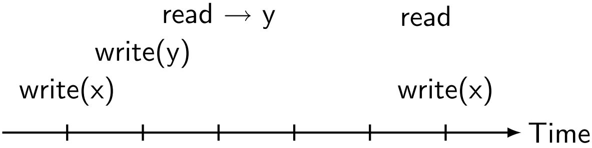
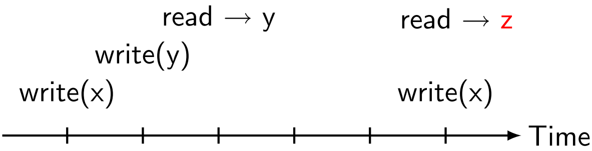
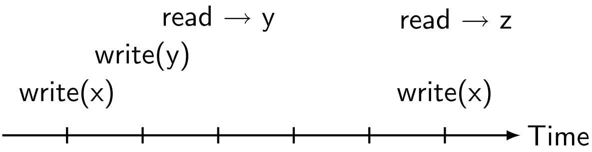



write
Safe register
read


read → y
read → z
write(y)
write(x)
write(x)
Time

Building blocks for the handshake register



- 4x safe register
- 4x atomic boolean register

Verifying the implementation

- Model the handshake register specification as a μCRL process
- Model the implementation as a μCRL process
- Generate their state spaces
- Minimise with respect to some equivalence ($\tau^* a$)
- Check for graph equivalence

Verifying the implementation

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Problem: state space explosion

Verifying the implementation

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- Model the implementation as a μCRL process
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- Check for graph equivalence

Problem: state space explosion

Solution: Apply `stategraph`! (and compare to `parelm`)

Applying stategraph

	constelm | parelm | constelm		
	states	time (expl.)	time (symb.)
$\lvert D\rvert = 2$	540,736	0:23.0	0:04.5
$\lvert D\rvert = 3$	13,834,800	10:10.3	0:06.7
$\lvert D\rvert = 4$	142,081,536	–	0:09.0
$\lvert D\rvert = 5$	883,738,000	–	0:11.9
$\lvert D\rvert = 6$	3,991,840,704	–	0:15.4

	constelm | stategraph | constelm		
	states	time (expl.)	time (symb.)
$\lvert D\rvert = 2$	45,504	0:02.4	0:01.3
$\lvert D\rvert = 3$	290,736	0:12.7	0:01.4
$\lvert D\rvert = 4$	1,107,456	0:48.9	0:01.6
$\lvert D\rvert = 5$	3,162,000	2:20.3	0:01.8
$\lvert D\rvert = 6$	7,504,704	5:26.1	0:01.9

Why the reduction was obtained

$$
\begin{array}{llllr}
\multicolumn{5}{l}{Y(i\colon \mathrm{Bool}, j\ \colon \mathrm{Bool}, r\colon \{1,2,3\}, w\colon \{1,2,3\}, v\colon D, vw\colon D, vr\colon D) =} \\
 & & r = 1 & \Rightarrow \mathrm{beginRead}(i,j) \cdot Y(i,j,2,w,v,vw,vr) & (1) \\
+ & & r = 2 \wedge w = 1 & \Rightarrow \tau \cdot Y(i,j,3,w,v,vw,v) & (2) \\
+ & \sum_{x\colon D} & r = 2 \wedge w \neq 1 & \Rightarrow \tau \cdot Y(i,j,3,w,v,vw,x) & (3) \\
+ & & r = 3 & \Rightarrow \mathrm{endRead}(i,j,vr) \cdot Y(i,j,1,w,v,vw,vr) & (4) \\
+ & \sum_{x\colon D} & w = 1 & \Rightarrow \mathrm{beginWrite}(i,j,x) \cdot Y(i,j,r,2,v,x,vr) & (5) \\
+ & & w = 2 & \Rightarrow \tau \cdot Y(i,j,r,3,vw,vw,vr) & (6) \\
+ & & w = 3 & \Rightarrow \mathrm{endWrite}(i,j) \cdot Y(i,j,r,1,vw,vw,vr) & (7)
\end{array}
$$

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\begin{array}{llll}
Y(i: \mathrm{Bool}, j & : \mathrm{Bool}, r: \{1,2,3\}, w: \{1,2,3\}, v: D, vw: D, \color{red}{vr: D}) = & & \\
 & r = 1 & \Rightarrow \mathrm{beginRead}(i,j) \cdot Y(i,j,2,w,v,vw,vr) & (1) \\
+ & r = 2 \wedge w = 1 & \Rightarrow \tau \cdot Y(i,j,3,w,v,vw,\color{blue}{v}) & (2) \\
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Other case studies

Other specifications `stategraph` was applied to:

- An Automatic In-flight Data Acquisition unit for a helicopter
- A cache coherence protocol for a distributed JVM
- The sliding window protocol
- An automatic translation from Erlang to μCRL of a distributed resource locker in Ericsson's AXD 301 switch

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Results:

- Reductions in the number of states (up to 20 percent)
- Reductions in the number of parameters (up to 75 percent)
- Reductions in the number of summands (up to 25 percent)

Contents

Conclusions

- Novel method for reconstructing control flow
  - Even control flow hiding in state parameters is found
- Data flow analysis based on this control flow
  - Resetting variables that are no longer relevant
  - Decreases in states, parameters and summands
  - Reductions obtained before generating the entire state space
- Precise proofs of correctness *and* decrease of state space
- Case studies show that impressive results are indeed obtained

Future work

- Investigate additional applications for the reconstructed control flow
    - Invariant generation
    - Visualisation (already implemented)
    - Improve confluence checking
- Use more precise abstractions based on control flow

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- Investigate additional applications for the reconstructed control flow
  - Invariant generation
  - Visualisation (already implemented)
  - Improve confluence checking
- Use more precise abstractions based on control flow
- Apply these techniques to a probabilistic linear format (currently in development)

Questions

?