UNIVERSITY OF TWENTE.

Formal Methods & Tools.

Confluence versus Partial Order Reduction in Statistical Model Checking and Simulation

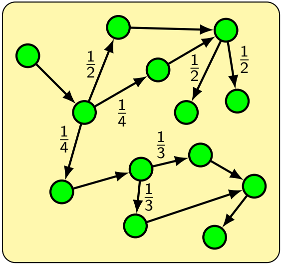
Mark Timmer

September 13, 2012

Aachen 2012

Joint work with Arnd Hartmanns

(Statistical) Model Checking



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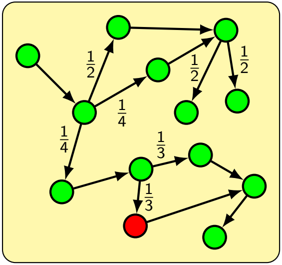

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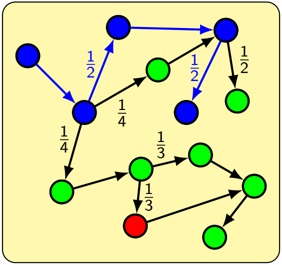

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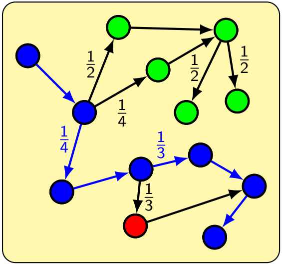

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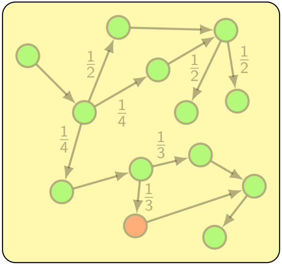

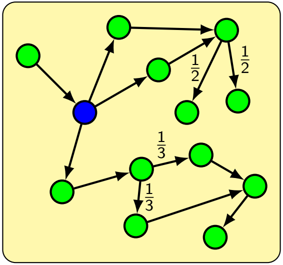

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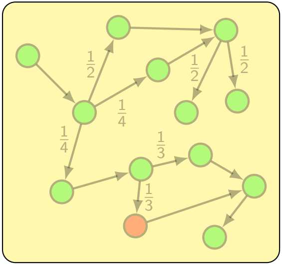

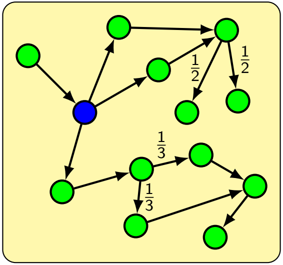

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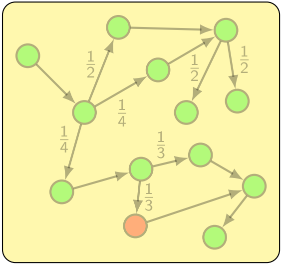

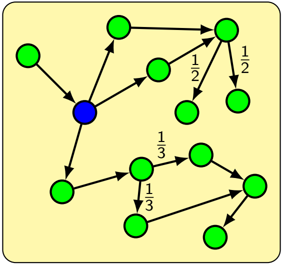

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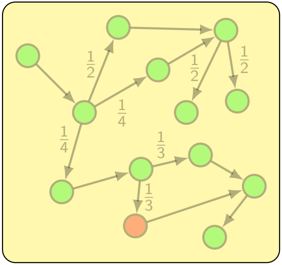

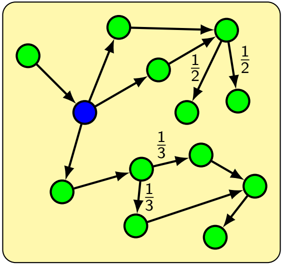

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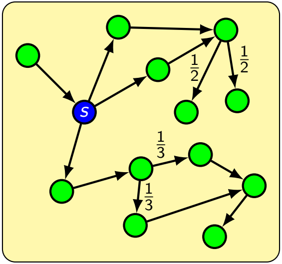

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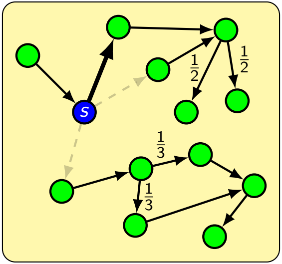

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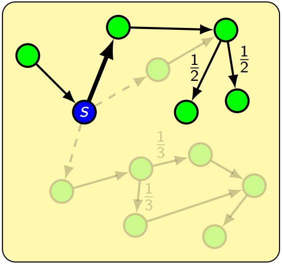

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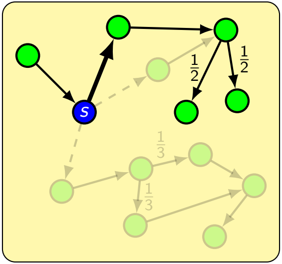
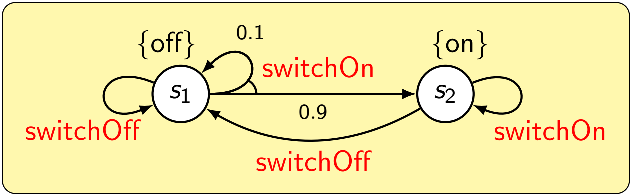


Important:

reduced system should be equivalent

The model: Probabilistic Automata

Probabilistic Automaton:



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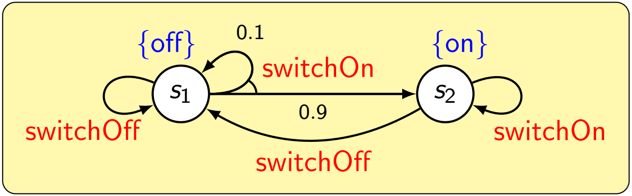

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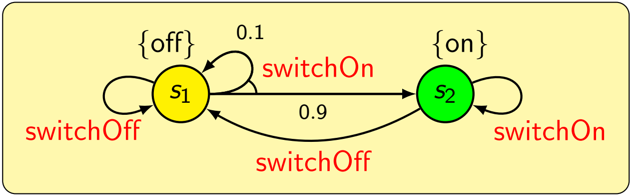

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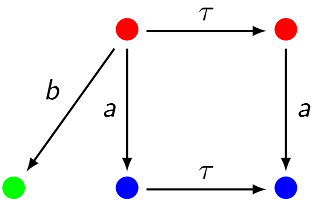

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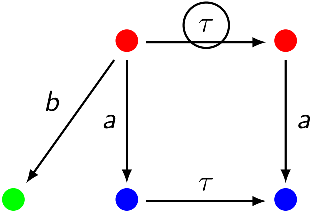

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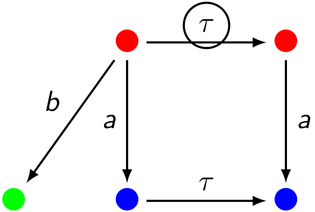

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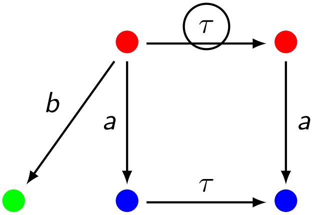

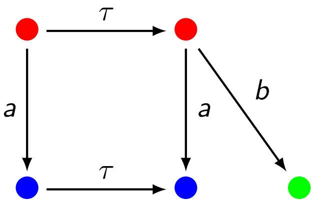

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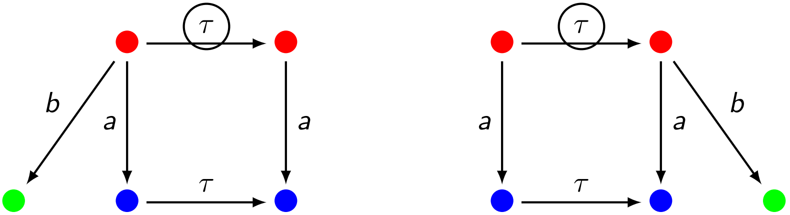

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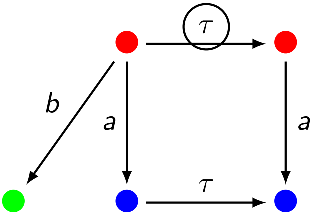

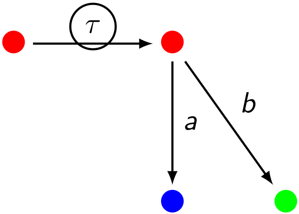

Non-probabilistic and probabilistic confluence reduction

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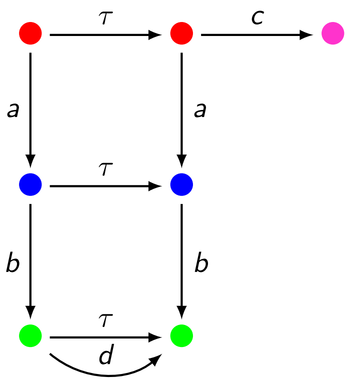

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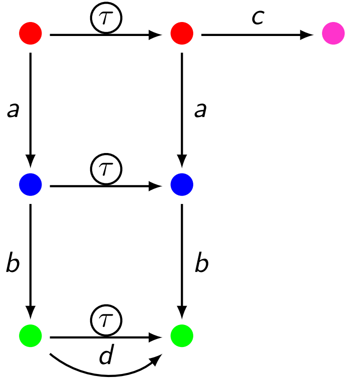

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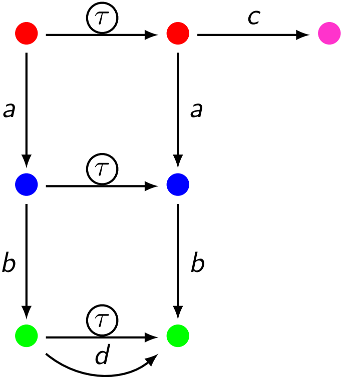

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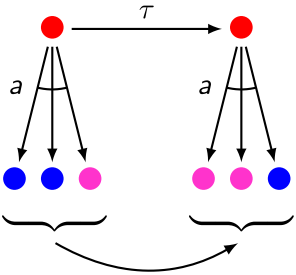

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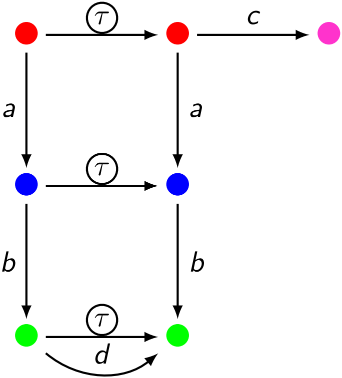

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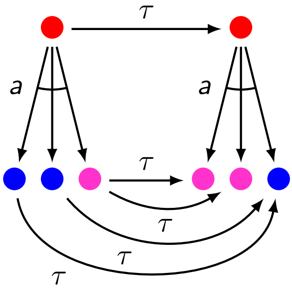

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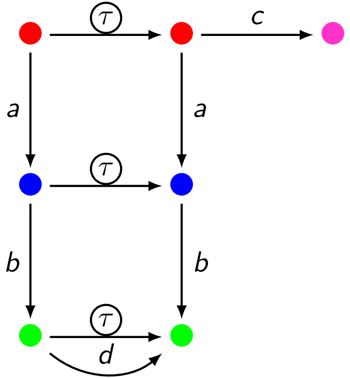

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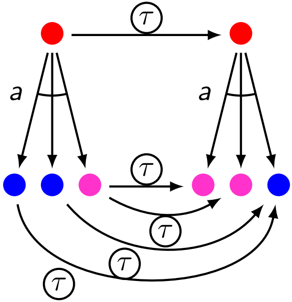

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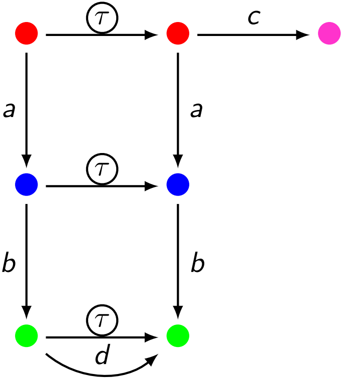

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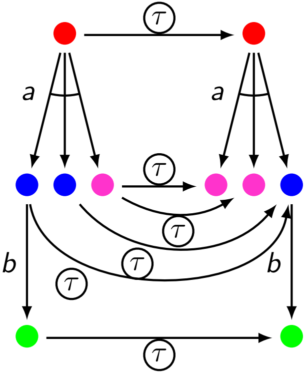

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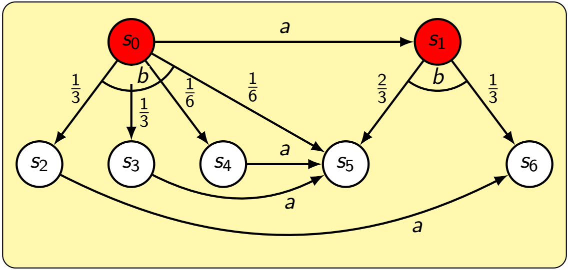

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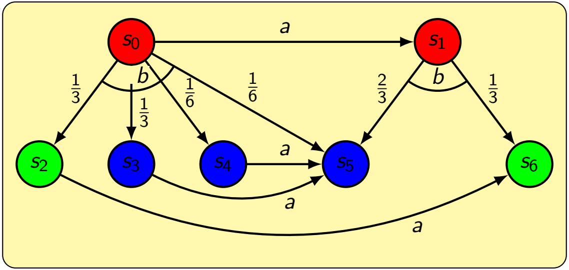

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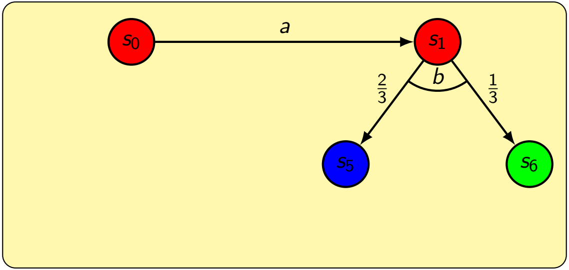

Alterations to the concept of confluence

- Transitions may be mimicked by differently-labelled transitions
- Transitions only have to be invisible locally
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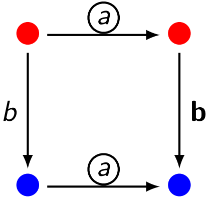

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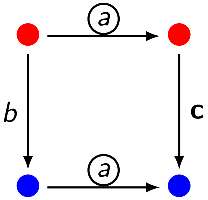

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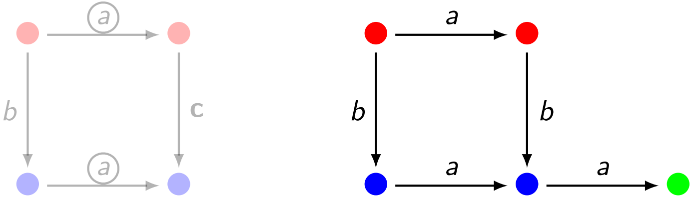

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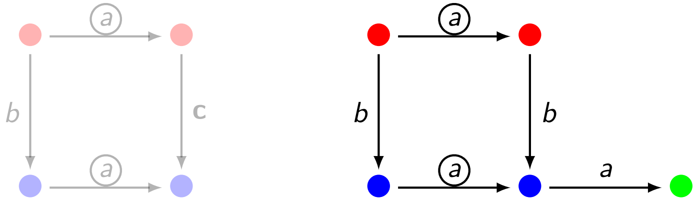

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Distributions μ and ν are $\mathcal{T}$-equivalent, if there exists a partitioning $\mathrm{spt}(\mu) = \biguplus_{i=1}^{n} S_i$ of the support of μ and an ordering $\mathrm{spt}(\nu) = \{s_1, \dots, s_n\}$ of the support of ν, such that $\forall 1 \leq i \leq n$

$$\mu(S_i) = \nu(s_i) \wedge (S_i = \{s_i\} \vee \forall s \in S_i \,.\, \exists a \in \Sigma \,.\, s \xrightarrow{a} s_i \in \mathcal{T}).$$

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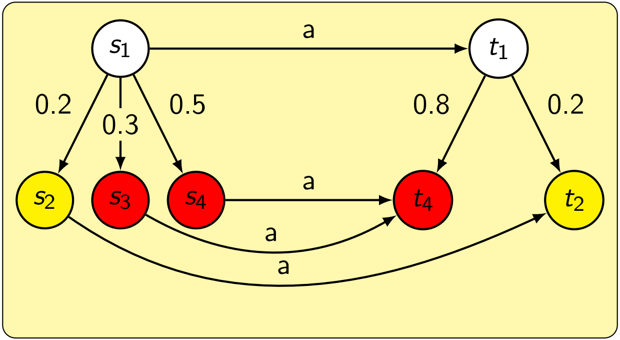

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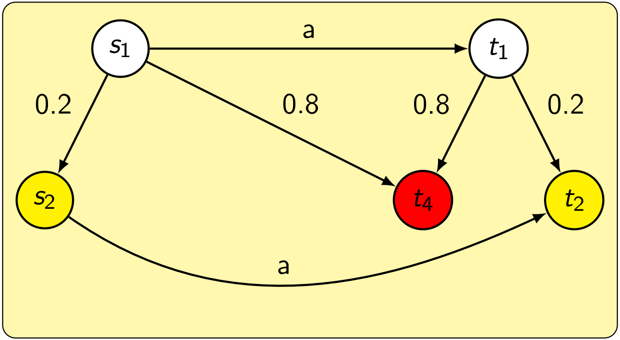

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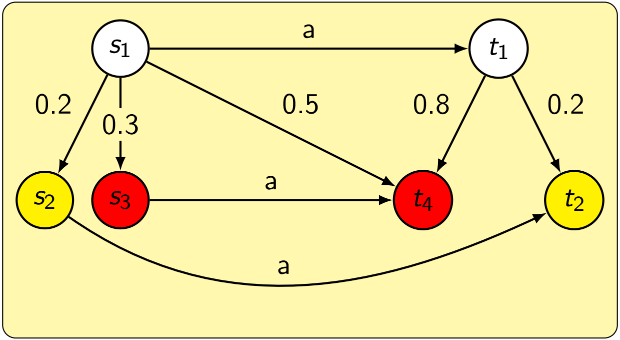

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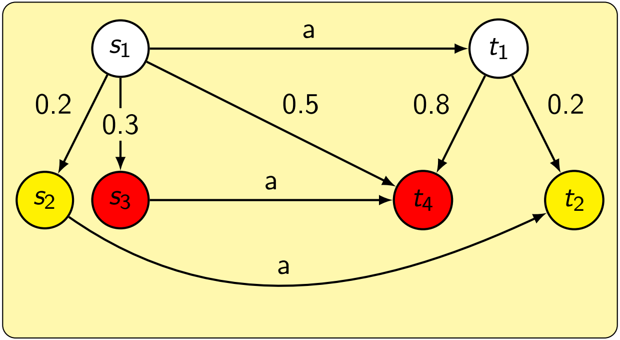

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Still we find:

Theorem

Confluent transitions can be given priority, preserving $PCTL^*_X$.

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Simulation using on-the-fly confluence detection:

1. Simulate until reaching a nondeterministic choice

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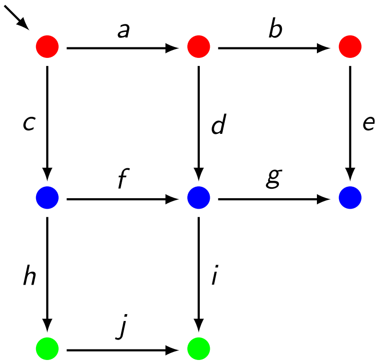
- Check if all its neighbouring transitions are mimicked
  - For this, additional transitions might need to be confluent

Checking a transition for confluence



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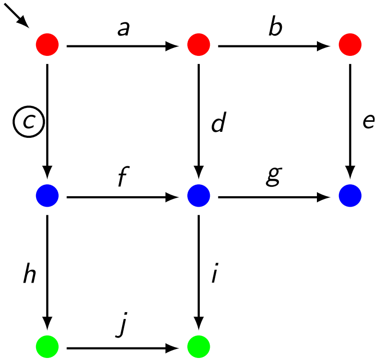

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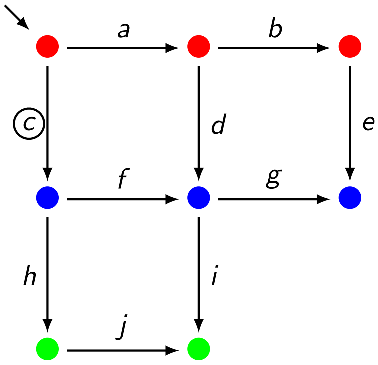
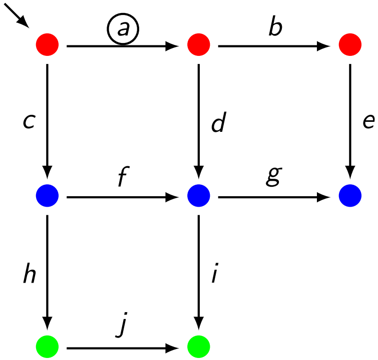
Checking a transition for confluence



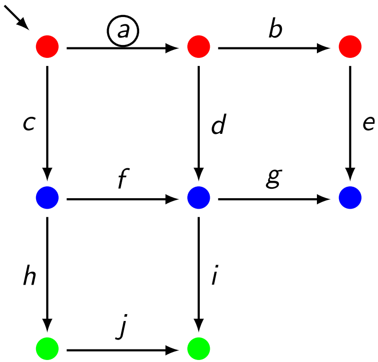
- Check if c is confluent
  - No; it is not invisible

Checking a transition for confluence



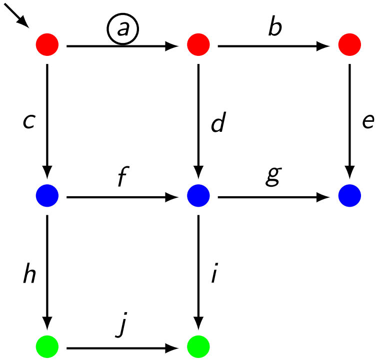
- Check if c is confluent
  - No; it is not invisible
- Check if a is confluent

Checking a transition for confluence



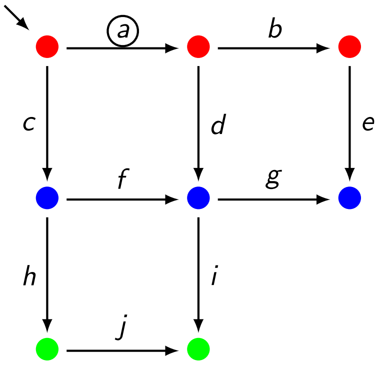
- Check if c is confluent
  - No; it is not invisible
- Check if a is confluent
  - It is invisible

Checking a transition for confluence



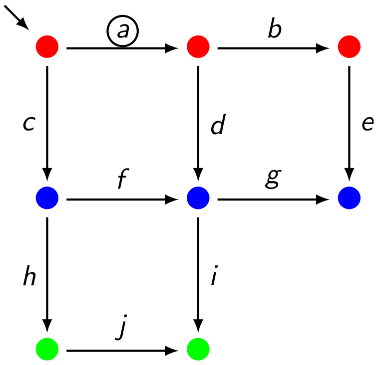
- Check if c is confluent
  - No; it is not invisible
- Check if a is confluent
  - It is invisible
  - Is the c-transition mimicked?

Checking a transition for confluence



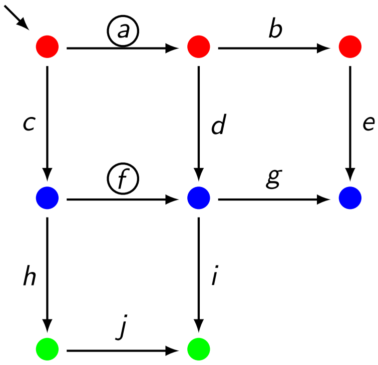
- Check if *c* is confluent
  - No; it is not invisible
- Check if *a* is confluent
  - It is invisible
  - Is the *c*-transition mimicked?
    - Possibly by the *d*-transition

Checking a transition for confluence



- Check if c is confluent
  - No; it is not invisible
- Check if a is confluent
  - It is invisible
  - Is the c-transition mimicked?
    - Possibly by the d-transition
    - But then f has to be confluent: check this
    - ...

Checking a transition for confluence



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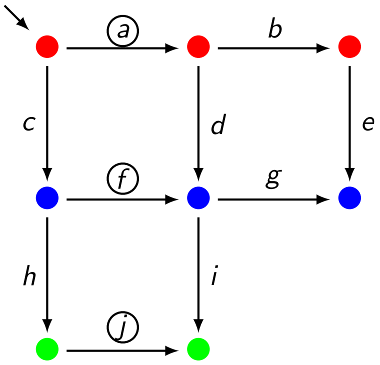

- Check if c is confluent
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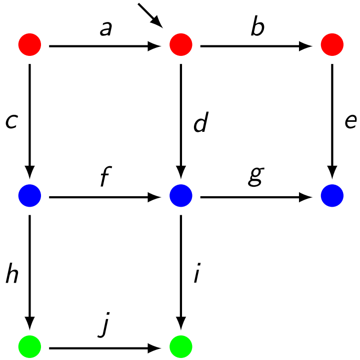

- Check if c is confluent
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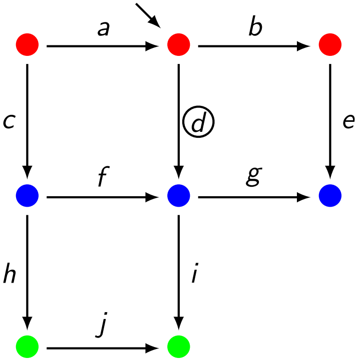
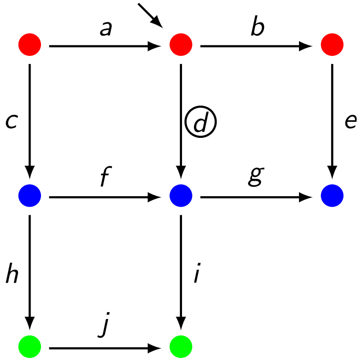


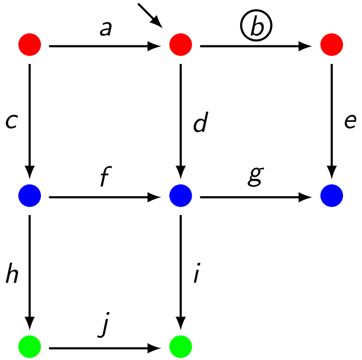
- Check if d is confluent

Checking a transition for confluence



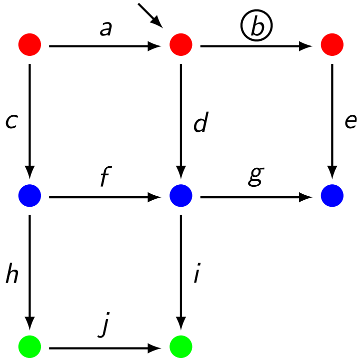
- Check if *d* is confluent
  - No; it is not invisible

Checking a transition for confluence



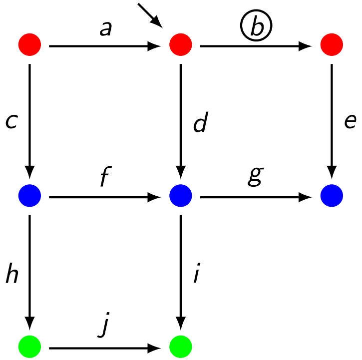
- Check if d is confluent
  - No; it is not invisible
- Check if b is confluent

Checking a transition for confluence



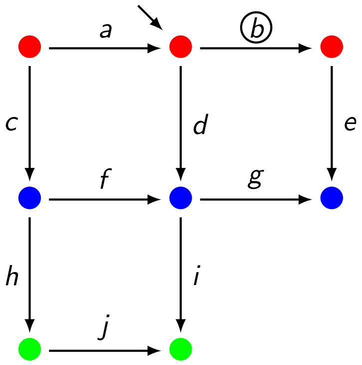
- Check if d is confluent
  - No; it is not invisible
- Check if b is confluent
  - It is invisible

Checking a transition for confluence



- Check if *d* is confluent
  - No; it is not invisible
- Check if *b* is confluent
  - It is invisible
  - Is the *d*-transition mimicked?

Checking a transition for confluence



- Check if *d* is confluent
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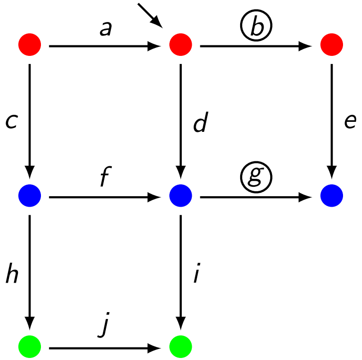
  - Is the *d*-transition mimicked?
    - Possibly by the *e*-transition

Checking a transition for confluence



- Check if d is confluent
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    - But then g has to be confluent: check this
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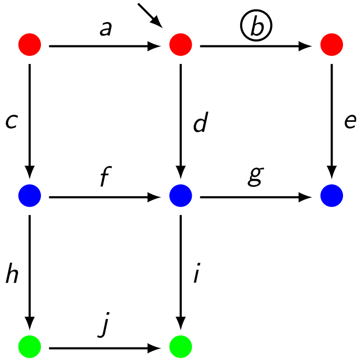

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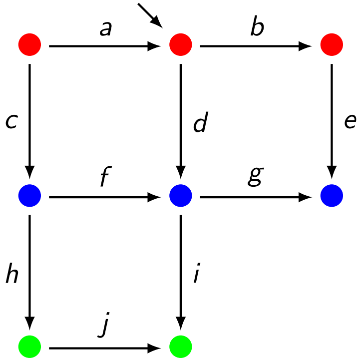

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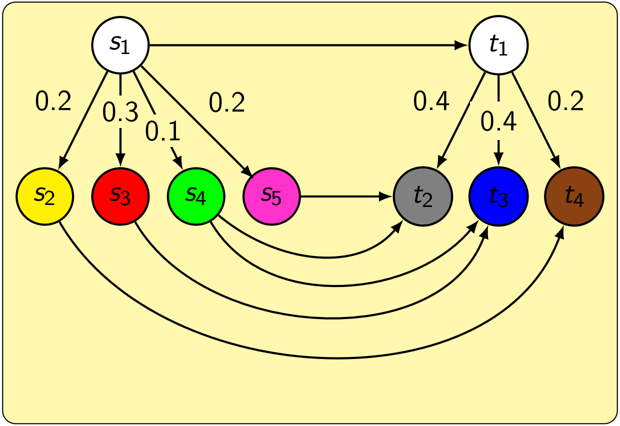
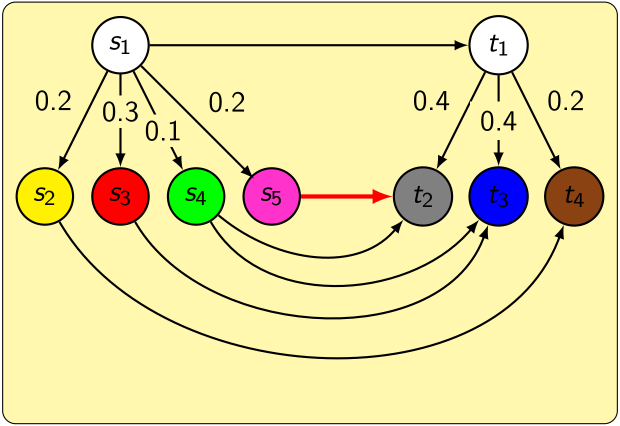
Detecting equivalence of transitions



$$\boxed{\{(s,t) \mid s \in \mathrm{spt}(\mu), t \in \mathrm{spt}(\nu), s \xrightarrow{a} t \in \mathcal{T}\}}$$

Detecting equivalence of transitions



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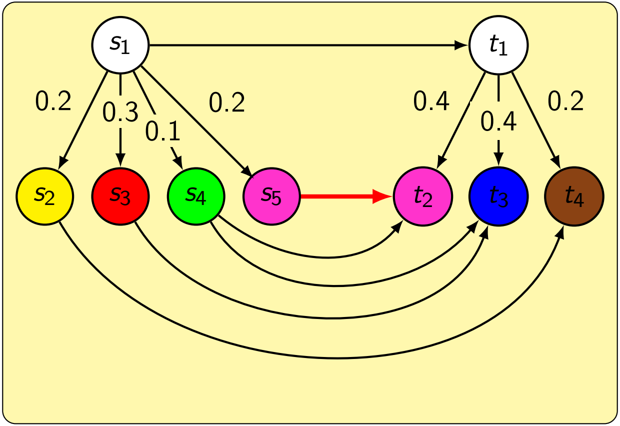

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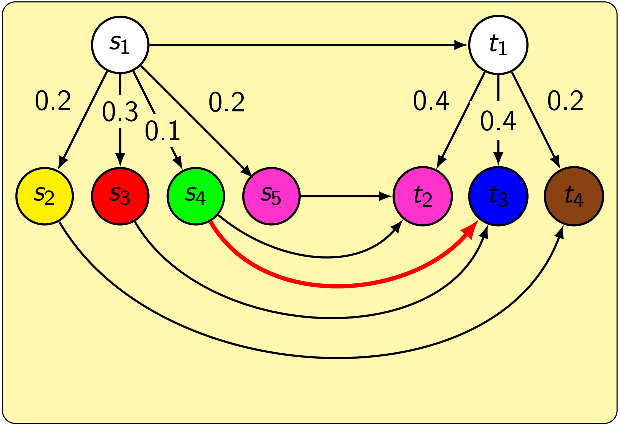

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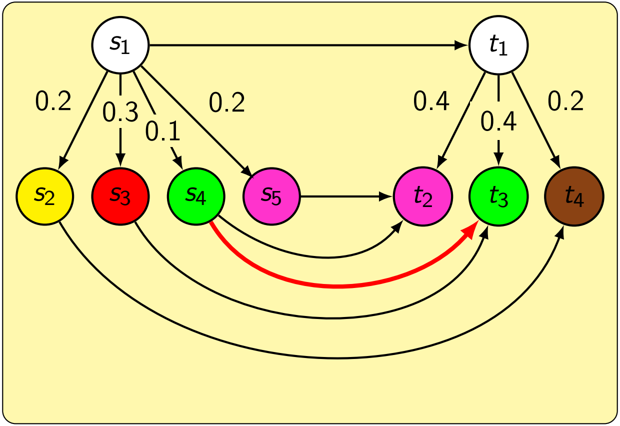

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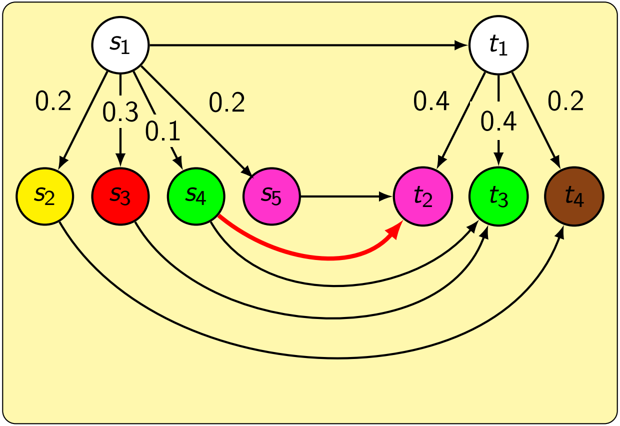

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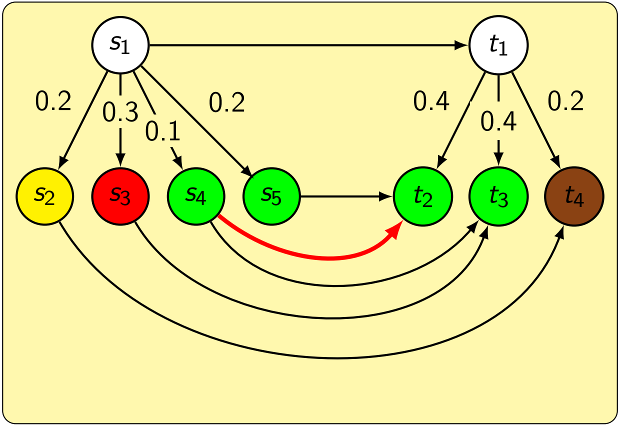

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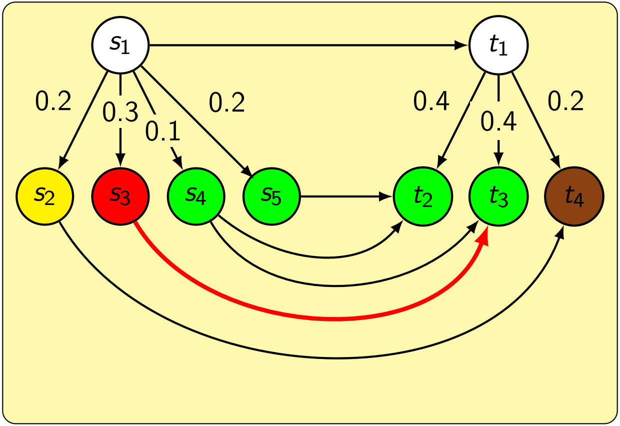

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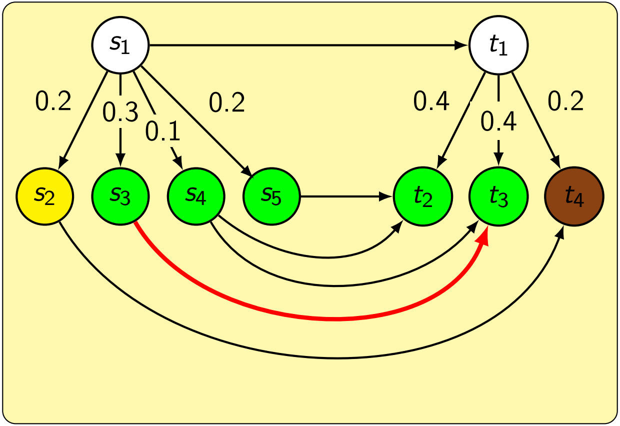

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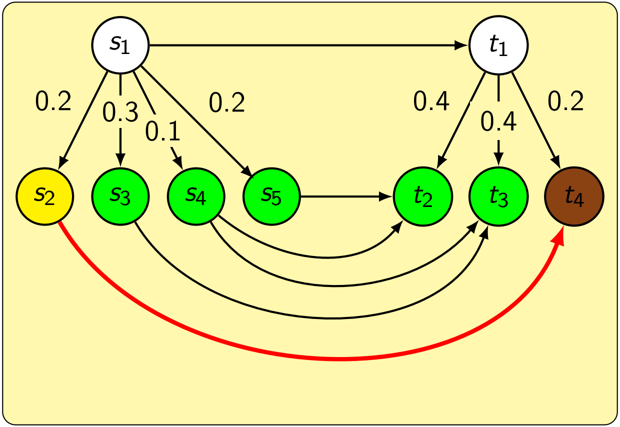

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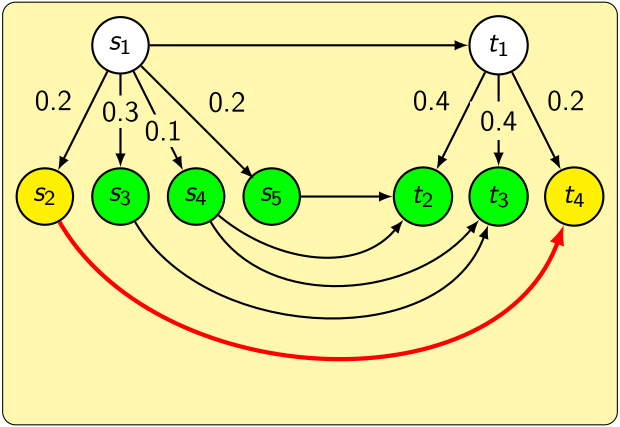

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Implementation

modes: a discrete-event simulator for the MODEST language

- Statistical model checking of deterministic systems
  - Partial order reduction
  - Confluence reduction

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Three case studies:

- Dining Cryptographers
- IEEE 802.3 CSMA/CD
- Binary Exponential Backoff

Case study: Dining Cryptographers

Table : Confluence simulation runtime compared

model	simulation	
(N)	uniform	confluence
3	3 s	13 s
4	4 s	66 s
5	5 s	338 s

Partial order reduction was not able to resolve the nondetermism.

Case study: CSMA/CD

Table : Confluence simulation runtime compared

model	simulation		model checking	
(RED, BC_{MAX})	uniform	confluence	states	time
$(2, 1)$	6 s	18 s	15283	11 s
$(1, 1)$	6 s	18 s	30256	51 s
$(1, 2)$	11 s	48 s	194818	214 s

Partial order reduction was not able to resolve the nondetermism.
(for confluence, probabilistic transitions needed to be synchronised)

Case study: Binary Exponential Backoff

Table : Confluence simulation runtime compared

model	simulation		
(K, N, H)	uniform	partial order	confluence
$(4, 3, 3)$	1 s	2 s	2 s
$(8, 7, 4)$	14 s	18 s	16 s

Conclusions

- We improved on the notion of confluence reduction:
  - Independent of action labels
  - Independent of global behaviour
  - More liberal equivalence of distributions

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Conclusions

- We improved on the notion of confluence reduction:
  - Independent of action labels
  - Independent of global behaviour
  - More liberal equivalence of distributions
- We provided an on-the-fly detection algorithm for SMC
- We implemented the new technique in MODEST
- Case studies show that confluence reduction reduces more and slightly faster than partial order reduction
- More models can now statistically be checked

Questions

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